THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH2050 (First Term)
Mathematical Analysis I
Homework II

Questions with * will be marked.

1. Let $A := \{x \in \mathbb{R} : 0 < x \leq 1\}$, i.e. $A = (0,1]$. Find $\max A$, $\inf A$, $\max A$ and $\sup A$ (if exist). Give your reasoning (including for your non-existence claim).

2. * Do the same for the set
$$S := \left\{ \frac{1}{n} - \frac{1}{m} : m, n \in \mathbb{N} \right\}.$$

3. * Let f, g be real-valued bounded above functions on a set X. Show that
$$\sup\{f(x) + g(x) : x \in X\} \leq \sup\{f(x) : x \in X\} + \sup\{g(x) : x \in X\},$$
or, in convenient notations
$$\sup_{x \in X}(f(x) + g(x)) \leq \sup_{x \in X} f(x) + \sup_{x \in X} g(x).$$
Can the strict inequality (or equality) happen?

4. * Let (x_n) be a sequence converge to a real number x. Show by $(\varepsilon - N)$ definition, that
   (a) $\lim_{n \to \infty} |x_n| = |x|$;
   (b) if $\alpha < x < \beta$ then there exists $N \in \mathbb{N}$ such that $\alpha < x_n < \beta$ for all $n \geq N$. (Hint: consider $\varepsilon_0 := \min\{\beta - x, x - \alpha\}$.)

5. Let $z < x < y$ and $0 < \ell < y - x$. Show that there exists $\bar{m} \in \mathbb{N}$ such that $x < z + \bar{m}\ell < y$. (Hint: See Appendix.)