UGED 1533 - Mathematics in Visual Art

Week 2

Proportions

2.1 Human Proportions

Proportional measurement is a useful tool for drawing objects (in particular the human figure) to scale. But besides this practical aspect, artists since Ancient Greece have incorporated into the aesthetic of their work the idea of **ideal proportions**.

The practice & science of drawing

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The practice & science of drawing, by Harold Speed.

Bridgman - Complete Guide to Drawing from Life.pdf (PDFy mirror)

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Complete Guide to Drawing from Life, by George Bridgman.

Figure Drawing for All It's Worth

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Figure Drawing for All It's Worth, by Andrew Loomis.

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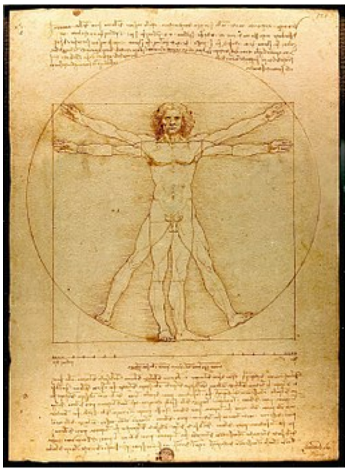
To the general public, one of the most famous examples of the study of ideal proportions of the human body is without doubt Leonardo da Vinci's "Vitruvian Man", named after the Roman architect and engineer Marcus Vitruvius Pollio, who lived in the 1st century BCE.



"The Vitruvian Man" (c. 1492)
Leonard da Vinci

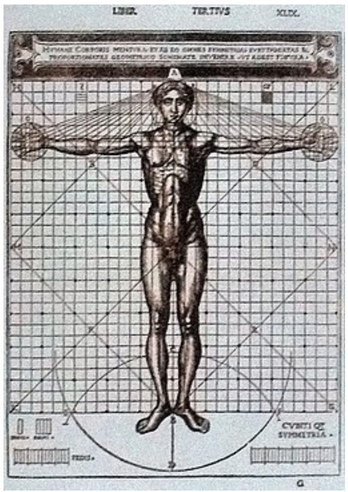


Illustration in a 16th-century edition of Vitruvius's *De Architectura*.

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For the human body is so designed by nature that the face, from the chin to the top of the forehead and the lowest roots of the hair, is a tenth part of the whole height; the open hand from the wrist to the tip of the middle finger is just the same; the head from the chin to the crown is an eighth, and with the neck and shoulder from the top of the breast to the lowest roots of the hair is a sixth; from the middle of the breast to the summit of the crown is a fourth. If we take the height of the face itself, the distance from the bottom of the chin to the under side of the nostrils is one third of it; the nose from the under side of the nostrils to a line between the eyebrows is the same; from there to the lowest roots of the hair is also a third, comprising the forehead. The length of the foot is one sixth of the height of the body; of the forearm, one fourth; and the breadth of the breast is also one fourth. The other members, too, have their own symmetrical proportions, and it was by employing them that the famous painters and sculptors of antiquity attained to great and endless renown.

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Who are these "famous painters and sculptors of antiquity"? Two of the most famous Greek sculptors from Classical Antiquity are Phidias (who supervised the building of the Parthenon in Athens) and Polykleitos. Apelles was considered one of the greatest painters of all time.

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The example we shall examine here is the **Doryphoros** ("Spear Bearer") statue, by the 5th-century BCE Greek sculptor **Polykleitos** . The original bronze sculpture has not survived to the present day. All we have are various Roman marble copies.

Altes Museum, Berlin, Germany

Naples National Archaeological Museum, Naples, Italy

The Cinquantenaire Museum, Brussels, Belgium

Various copies of the Doryphoros Statue

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We have little first-hand information regarding the life and personal philosophy of Polykleitos. From secondary sources such as the Greek medical writer Galen (2nd century CE), it is believed that Polykleitos believed in certain ideal proportions of the human figure. These views were supposedly recorded in a written treatise of his titled "**Kanon**", which means "rule" in Greek, and often anglicized to "Canon". It is believed that the original bronze Doryphoros sculpture was made to demonstrate the Kanon, and for this reason the status itself was also called "Kanon" by its creator.

The problem is that no one seems to know what exactly are the ideal proportions according Polykleitos. An oft-quoted study on the proportions exemplified by the Doryphoros statue is a paper titled *The Canon of Polykleitos*, by Richard Tobin, which was published in the American Journal of Archaeology in 1975[1].

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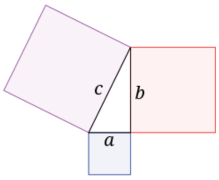
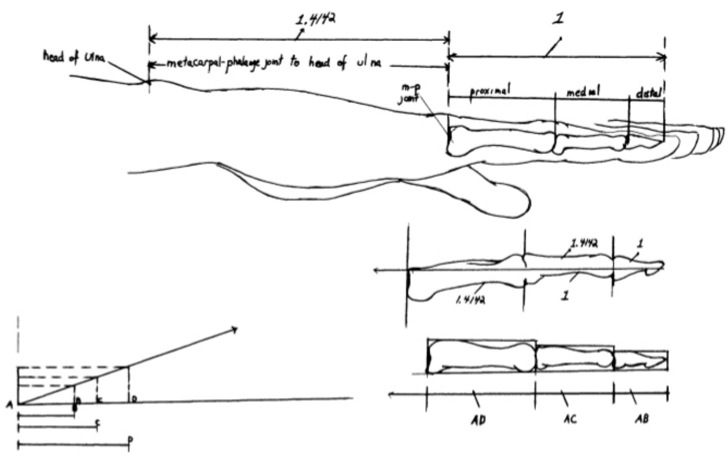
Tobin proposes that Polykleitos used the distal phalanx (the tip segment) of the little finger as the fundamental length unit, "Having determined 'from nature' its length and width".

The length of each subsequent "segment" of the human body (the Intermediate phalanges, "the palm-wrist, forearm, upper arm") is equal to the length of the previous segment times $\sqrt{2}$. This can be done geometrically by drawing a square whose side length is the length of the previous segment, and by the Pythagorean theorem the length of the subsequent segment is just the length of the diagonal of the square. Here, we are using the word segment loosely.

For example, when determining the length of each phalanx on a finger, Tobin's proposed system uses the preceding (from small to large) phalanx as the previous segment. But: "On the hand itself, the three phalanges of the little finger must be taken together so that their total length can yield the palm-wrist length." In other words, the definition of "segment" depends on the part of the arm we are working with.



Pythagorean Theorem
$a^2+b^2=c^2$



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Richard Tobin's illustration of his reconstruction of Polykleitos's *Kanon*.

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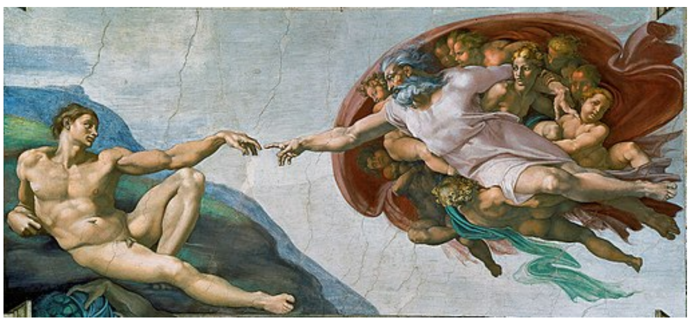
Moving from the arm to the body proper, a slightly different system is adopted.
According to Tobin, Polykleitos "shifts the upper arm length over to the body, where it becomes the distance from the head to the clavicle, along the imaginary line between the acromia on the top of the shoulders."
From this length we again obtain a progression of lengths, each obtained from the preceding one via scaling by $\sqrt{2}$. Each such length is equal to the distance between a reference point (shoulders, nipples, abdomen, groin, knees, base of the feet) on the human body and *the top of the head*(instead of the end of the previous segment, as in the case of the arm).
Then, Tobin claims that Polykleitos, working backwards, obtained that the length of the head is equal to the length of the forearm, so that the length from the top of the head to the acromia line also satisfies the $\sqrt{2} : 1$ ratio to the length of the head. (Your instructor is puzzled why Tobin didn't simply propose that Polykleitos first determined the length of the head using the length of the forearm, then worked forward from which point on.)
If we are to believe that this is indeed the system which Polykleitos adopted, then the ideal ratio between the height of a man and the length of his head is:

$$1 : (\sqrt{2})^6 = 1 : 8.$$

In other words, under this proposed system, the height of the ideal human physique measures 8 heads in length. Interestingly, that is the proportion adopted by the Renaissance artist Michelangelo Buonarroti (at least according to George Bridgman).



Creation of Adam (1508 - 1512)
Michelangelo Buonarroti

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2.2 Comparative Measurements in Drawing

When using comparative measurement in drawing (say of the human figure), the student may find that certain lengths is not an exact integral multiple of the the length of the measuring unit.
For example, should we choose our measuring unit to be the distance between the bottom of the chin to the top of the head. Then, we may encounter a model where the vertical distance between

the top of the shoulders and the top of the head appears to be about one and a half.

For the student with an obsessive bent toward accuracy (most artists have a certain degree of obsessive bent after all), one way to make sure is to reduce our fundamental measuring unit to one half of the head. This is often approximately the distance between the brow line and the top of the head. Then, the length of the head is 2 times this new unit, and, if we are lucky, the distance between the top of the shoulders and the top of the head might turn out to be exactly 3 times the fundamental unit.

This solution makes our measurements somewhat easier, for intermediate lengths like 2/3, 1/2 or 3/5 of the measuring unit may not seem so distinguished from one another to the human eye.

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There is a mathematical notion involved here, namely, that of **commensurability**. This notion was familiar to ancient Greek mathematicians such as the Pythagoreans (followers of the teaching/philosophy of Pythagoras) and Euclid. They did not have a definition of real numbers like we do now, so they often formulated their ideas on numbers in geometric terms. The formulation of commensurability in Euclid's *Elements* is one such example.

To Euclid, two line segments a, b are said to be commensurable if there is a third line segment c such that both a and b may be obtained by laying c end-to-end on a straight line a whole number of times.

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Translating to a more modern math lingo, we replace geometric object line segment with the number which is its length, and say that:

*Two numbers a, b are commensurate if there is a number c and whole numbers m, n such that $a = mc$ and $b = nc$.*In modern terms, the commensurability of two numbers a and b is equivalent to the statement that the fraction $\frac{a}{b}$ is a rational number, namely the rational number represented by $\frac{m}{n}$.

Taking the special case $b = 1$, we see that a real number a is rational if and only if it is commensurate with 1.

One of the first real numbers known to be irrational is $\sqrt{2}$. The first proof of the irrationality of $\sqrt{2}$ is attributed to a Pythagorean, possibly Hippasus of Metapontum, who lived in the 5th century BCE.

To many casual math students nowadays, $\sqrt{2}$ is the real number represented by the non-terminating decimal expansion:

$$1.4142135623...$$

But to the ancient Greeks, it was understood to be the hypotenuse of the right isosceles triangle whose remaining sides both have unit length 1. It can be shown that this hypotenuse is not commensurable with either side of the triangle, which is equivalent to the modern formulation that $\sqrt{2}$ is irrational.

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Going back the proportions of the human body, it is interesting to note that to Vitruvius various lengths associated to the body are commensurate with one another. On the other hand, in Tobin's proposed Kanon for the Doryphoros statue, various body parts are not necessarily commensurate with one another, since $\sqrt{2}$ is irrational.

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2.3 Dynamic Symmetry

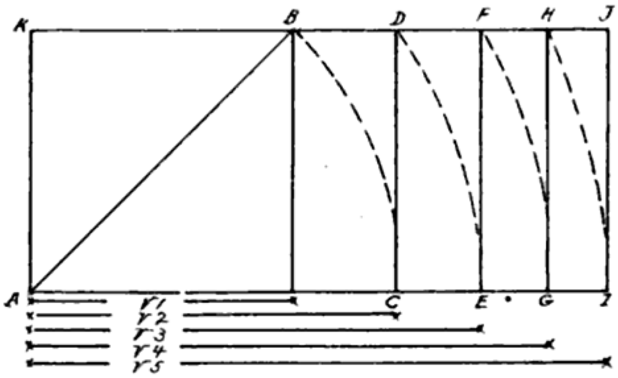

Fig. 10.

Root Rectangles

The proposed use of the $1 : \sqrt{2}$ ratio in the Doryphoros statue is an example of a broader type of design system called **dynamic symmetry** promoted by American artist Jay Hambidge (1867-1924). Hambidge proposed that the designs of classical Greek architecture, sculpture and ceramics made prominent use of proportions obtained from so-called "dynamic rectangles" which, starting with a square with sides of unit length 1, are obtained recursively using basic geometric constructions.

A basic example is series of "root rectangles", which all have unit height 1, and those sides are of lengths:

$$1, \sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots.$$

For example, the $1 : \sqrt{2}$ ratio corresponds to the root rectangle which is constructed as follows: Start with a unit square, draw a diagonal line joining one corner to another, then by means of a compass form a horizontal line which has the same length (i.e. $\sqrt{2}$) as the diagonal line. The rectangle whose base side is this horizontal line and those vertical side has unit length one is the root rectangle corresponding to the ratio $1 : \sqrt{2}$, for that is precisely the ratio of the lengths of its vertical and horizontal sides.

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Note that the use of dynamic symmetry in Classical Greece is a *theory*proposed by Hambidge, who apparently did not hold much enthusiasm for the rational proportions of Vitruvius, of whom he wrote:

The Roman architect had no knowledge of symmetry beyond a crude form of the static. He declared that the Greeks used a module to determine the symmetry of their temples and gives most elaborate instructions as to how the plans were developed. No Greek design has been found which agrees with the Vitruvian statements. In fact, the module would produce a grade of static symmetry which would have afforded much amusement to a Greek.[2]

We will for the time being refrain from weighing in on Hambidge's appraisal of Vitruvius's work, but we would like to use Hambidge's idea to pivot to our next topic: