MATH1050 Answers to Examples: Set notations and method of specification.

1. (a) $0, 1, 2, 3$.
   (b) $0, 1, 2, 3, 4$.
   (c) $0, 1, 2, \{0\}, \{1\}$.
   (d) $0, 1, \{0, 1\}$.
   (e) $0, 1, \{0, 1\}, \{\{0, 1\}\}$.

2. (a) $0, 1, 2, 3$.
   (b) $0, 1, 2$.
   (c) $0, 1, 3$.
   (d) $0, 2$.
   (e) $0, 1, 2, 3, \{1\}, \{2, 3\}$.
   (f) $0, 1, 2, 3, \{1, 2\}, \{3\}$.
   (g) $1, 2, 3, \{1, 2\}, \{3\}$.
   (h) 2.
   (i) $\{2, 3\}$.
   (j) $\{2\}, \{3\}$.
   (k) $\{1, 2\}$.
   (l) $\emptyset$, $\{0\}$, $\{2\}$, $\{\{1\}\}$, $\{0, 2\}$, $\{0, \{1\}\}$, $\{2, \{1\}\}$, $\{0, 2, \{1\}\}$.

3. (a) $0, 1, 2, 3, 4$.
   (b) $0, 1, \{1, 2, 3\}, \{\{3\}, 4\}$.
   (c) $0, 1$.
   (d) $0, 1, 2, 3, 4, \{1, 2, 3\}, \{\{3\}, 4\}$.
   (e) $0, 1, 2$.
   (f) $\{1, 2, 3\}$, $\{\{3\}, 4\}$.
   (g) $0, 1, 2, \{1, 2, 3\}$, $\{\{3\}, 4\}$.
   (h) $\emptyset$, $\{0\}$, $\{1\}$, $\{0, 1\}$.

4. (a) $\{0, 1\}$, $\{1, 2, 3\}$.
   (b) $\{0, 1\}$, $\{1\}$, $\{1, 2, 3\}$, $\{3, 4\}$, $\{\{3\}, \{4\}\}$.
   (c) $\{1\}$, $\{3, 4\}$.
   (d) $\{1\}$,$\{3, 4\}$, $\{\{3\}, \{4\}\}$.
   (e) $\emptyset$, $\{\{1\}\}$, $\{\{3, 4\}\}$, $\{\{1\}, \{3, 4\}\}$.

5. (a) $\{3, 5\}$.
   (b) $\{3, 5\}, \{5, 7\}, 1, \{5\}$.
   (c) $\{5, 7\}$.
   (d) $\emptyset, \{\{3, 5\}\}, \{\{5, 7\}\}, \{\{3, 5\}, \{5, 7\}\}$.

6. (a) $\{h\}, \{n\}$.
   (b) $\{b, e\}, \{e\}, \{t\}, \{h\}, \{o, v\}, \{n\}, \{a, y, d\}$.
   (c) $\{e\}, \{o, v\}, \{n\}$.
   (d) $\emptyset, \{\{m, o\}\}, \{\{z, a, r, t\}\}, \{\{m, o\}, \{z, a, r, t\}\}$.

7. (a) Ten.
   (b) Two.
   (c) One.
   (d) a, t.
   (e) $\emptyset$, $\{a\}$, $\{t\}$, $\{a, t\}$.

8. (a) Seven
   (b) Nine
   (c) Four
   (d) Seven
   (e) Two
   (f) One
   (g) s, u
   (h) $\emptyset, \{s\}, \{u\}, \{s, u\}$

9. (a) Ten.
   (b) One.
   (c) Four.
   (d) One.
   (e) u, o, d.
   (f) $\emptyset, \{u\}, \{o\}, \{d\}, \{u, o\}, \{o, d\}, \{u, d\}, \{u, o, d\}$.

10.

(a) Two

(b) Four

(c) Three

(d) $\emptyset$

(e) $\mathbb{N}$

(f) $\{\{\emptyset\}\}$, $\{\mathbb{N}\}$, G.

11. (a) $A = \emptyset$.

    (b) $B \neq \emptyset$; 8 is an element of B.

12. (a) $A = \emptyset$.

    (b) $B \neq \emptyset$; 8 is an element of B.

13. (a) True. (b) False. (c) False. (d) True. (e) False. (f) True.

14. (a) $\left\{x \in \mathbb{R} : x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$.

    Alternative answers: $\left\{x \in \mathbb{Q} : x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$, $\left\{x \in \mathbb{C} : x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$, $\left\{x \,\middle|\, x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$, ...

    Also acceptable: $\left\{\dfrac{1}{2^n} \,\middle|\, n \in \mathbb{N}\right\}$, ...

    Remark. In the expression '$\left\{x \in \mathbb{R} : x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$', the left-hand-side of the colon tells us from which 'larger set' $\mathbb{R}$ the elements of the constructed set are to be taken, while the right-hand-side of the colon tells us the 'condition' which the x's to be 'put inside' the constructed set has to satisfy exactly. This 'condition' is a 'predicate with variable x', so that whenever x is fixed it becomes a (mathematical) statement for which it makes sense to tell whether it is true or false.

    It is wrong to write this expression as '$\left\{x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N} : x \in \mathbb{R}\right\}$'.

    Each of the following is also wrong:

    $\left\{x \mid x = \dfrac{1}{2^n} \text{ for any } n \in \mathbb{N}\right\}$, $\left\{x \mid x = \dfrac{1}{2^n} \text{ where } n \in \mathbb{N}\right\}$, $\left\{x \mid x = \dfrac{1}{2^n} \text{ and } n \in \mathbb{N}\right\}$, $\left\{x \mid x = \dfrac{1}{2^n}, n \in \mathbb{N}\right\}$.

    (b) $\left\{x \in \mathbb{R} : x = \dfrac{3^n}{5^n} \text{ for some } n \in \mathbb{N}\right\}$.

    (c) $\left\{x \in \mathbb{R} : x = \dfrac{2^m}{3^n} \text{ for some } m, n \in \mathbb{N}\right\}$.

    Alternative answers: $\left\{x \in \mathbb{Q} : x = \dfrac{2^m}{3^n} \text{ for some } m, n \in \mathbb{N}\right\}$, $\left\{\dfrac{2^m}{3^n} \,\middle|\, m \in \mathbb{N} \text{ and } n \in \mathbb{N}\right\}$, ...

    (d) $\{x \in \mathbb{R} : x = \pi^m - e^n \text{ for some } m, n \in \mathbb{N}\}$.