MATH1050 Examples: Set notations and method of specification.

1. *You are not required to justify your answer.*

   Consider each of the sets below. List every element of the set concerned, each exactly once.

   (a) $A = \{0, 1, 2, 3\}$.

   (b) $B = \{0, 0, 1, 2, 3, 1, 4\}$.

   (c) $C = \{0, 1, 2, \{0\}, \{1\}\}$.

   (d) $D = \{0, 1, \{0, 1\}\}$.

   (e) $E = \{0, 1, \{0, 1\}, \{\{0, 1\}\}\}$.

2. *You are not required to justify your answer.*

   Let $A = \{0, 1, 2, 3\}$, $B = \{0, 0, 2, 1, 0\}$, $C = \{1, 3, 3, 1, 0, 3\}$, $D = \{0, 2, \{1\}\}$, $E = \{0, \{1\}, \{2, 3\}\}$, $F = \{0, \{2\}, \{3\}\}$, $G = \{0, \{1, 2\}, \{3\}\}$.

   Consider each of the sets below. List every element of the set concerned, each element exactly once. Where the set concerned is the empty set, write '*this set is the empty set*'.

   (a) A (b) B (c) C (d) $B \cap D$ (e) $A \cup E$ (f) $A \cup G$ (g) $A \triangle G$ (h) $D \backslash E$ (i) $E \backslash D$ (j) $F \backslash E$ (k) $G \backslash F$ (l) $\mathfrak{P}(D)$

3. *You are not required to justify your answer.*

   Let $C = \{0, 1, 1, 2, 3, 3, 4\}$, $D = \{0, 1, \{1, 2, 3\}, \{\{3\}, 4\}\}$.

   Consider each of the sets below. List every element of the set concerned, each element exactly once. Where the set concerned is the empty set, write '*this set is the empty set*'.

   (a) C. (b) D. (c) $C \cap D$. (d) $C \cup D$. (e) $C \backslash D$. (f) $D \backslash C$. (g) $C \triangle D$. (h) $\mathfrak{P}(C \cap D)$.

4. *You are not required to justify your answer.*

   Let $C = \{\{0, 1\}, \{1\}, \{1, 2, 3\}, \{3, 4\}\}$, $D = \{\{0, 1, 1\}, \{1, 2, 3\}, \{\{3\}, \{4\}\}\}$.

   Consider each of the sets below. List every element of the set concerned, each element exactly once. Where the set concerned is the empty set, write '*this set is the empty set*'.

   (a) $C \cap D$. (b) $C \cup D$. (c) $C \backslash D$. (d) $C \triangle D$. (e) $\mathfrak{P}(C \backslash D)$.

5. *You are not required to justify your answer.*

   Let $A = \{\{3, 5, 3\}, 5, 7, 7\}$, $B = \{\{3, 5\}, \{5, 7\}\}$, $C = \{1, \{5\}\}$.

   Consider each of the sets below. List every element of the set concerned, each element exactly once. Where the set concerned is the empty set, write '*this set is the empty set*'.

   (a) $A \cap B$ (b) $B \cup C$ (c) $B \backslash A$ (d) $\mathfrak{P}(B)$

6. *You are not required to justify your answer.*

   Let $B = \{\{b, e\}, \{e\}, \{t\}, \{h\}, \{o, v\}, \{e\}, \{n\}\}$, $H = \{\{h\}, \{a, y, d\}, \{n\}\}$, $M = \{\{m, o\}, \{z, a, r, t\}\}$, $S = \{\{s, c\}, \{h\}, \{u\}, \{b, e\}, \{r\}, \{t\}\}$.

   Consider each of the sets below. List every element of the set concerned, each element exactly once. Where the set concerned is the empty set, write '*this set is the empty set*'.

   (a) $B \cap H$ (b) $B \cup H$ (c) $B \backslash S$ (d) $\mathfrak{P}(M)$

7. *You are not required to justify your answers in this question.*

   Let $C = \{c, a, n, t, o, r\}$, $D = \{d, e, d, e, k, i, n, d\}$, $K = \{k, r, o, n, e, c, k, e, r\}$.

(a) How many elements are there in the set $C \cup D$?

(b) How many elements are there in the set $\{C\} \cup \{D\}$?

(c) How many elements are there in the set $\{C \cup D\}$?

(d) List every element of the set $C \backslash K$, each element exactly once.

(e) List every element of the set $\mathfrak{P}(C \backslash K)$, each element exactly once.

8. *You are not required to justify your answers in this question.*

   Let $S = \{s, e, n, a, t, u, s\}$, $P = \{p, o, p, u, l, u, s, q, u, e\}$, $R = \{r, o, m, a, n, u, s\}$.

   (a) How many elements are there in the set P?

   (b) How many elements are there in the set $S \cup R$?

   (c) How many elements are there in the set $(S \cup R) \cap P$?

   (d) How many elements are there in the set $S \cup (R \cap P)$?

   (e) How many elements are there in the set $\{S\} \cup \{R \cap P\}$?

   (f) How many elements are there in the set $\{S \cup P\} \backslash \{R\}$?

   (g) List every element of the set $S \cap P \cap R$, each element exactly once.

   (h) List every element of the set $\mathfrak{P}(S \cap P \cap R)$, each element exactly once.

9. *You are not required to justify your answers in this question.*

   Let $Q = \{q, u, o, d\}$, $E = \{e, r, a, t\}$, $D = \{d, e, m, o, n, s, t, r, a, n, d, u, m\}$.

   (a) How many elements are there in the set D?

   (b) How many elements are there in the set $(Q \cup E) \backslash D$?

   (c) How many elements are there in the set $Q \cup (E \backslash D)$?

   (d) How many elements are there in the set $\{E\} \cup \{D \cap E\}$?

   (e) List every element of the set $Q \cap D$, each element exactly once.

   (f) List every element of the set $\mathfrak{P}(Q \cap D)$, each element exactly once.

10. *You are not required to justify your answers in this question.*

    Let $E = \{\emptyset, \mathbb{N}\}$, $F = \{\emptyset, \{\mathbb{N}\}\}$, $G = \{\{\emptyset\}, \mathbb{N}\}$, $H = \{\{\emptyset\}, \{\mathbb{N}\}\}$.

    (a) How many elements are there in the set E?

    (b) How many elements are there in the set $E \cup F \cup G \cup H$?

    (c) How many elements are there in the set $\{E, F, H\}$?

    (d) List every element of the set $E \cap F$, each element exactly once.

    (e) List every element of the set $G \backslash H$, each element exactly once.

    (f) List every element of the set $\mathfrak{P}(G) \backslash (E \cup G)$, each element exactly once.

11. *You are not required to justify your answers in this question.*

    Let $A = \{x \in \mathbb{N} \backslash \{0, 1\} : x^2 = n^3 \text{ for any } n \in \mathbb{Z}\}$, $B = \{x \in \mathbb{N} \backslash \{0, 1\} : x^2 = n^3 \text{ for some } n \in \mathbb{Z}\}$.

    (a) Is A the empty set? If *yes*, just write '$A = \emptyset$'. If *no*, write '$A \neq \emptyset$' and name one element of A.

    (b) Is B the empty set? If *yes*, just write '$B = \emptyset$'. If *no*, write '$B \neq \emptyset$' and name one element of B.

12. *You are not required to justify your answers in this question.*

    Let $A = \{x \in \mathbb{N} \backslash \{0\} : x = r^2 - r - 12 \text{ for any } r \in \mathbb{Z}\}$, $B = \{x \in \mathbb{N} \backslash \{0\} : x = r^2 - r - 12 \text{ for some } r \in \mathbb{Z}\}$.

    (a) Is A the empty set? If *yes*, just write '$A = \emptyset$'. If *no*, write '$A \neq \emptyset$' and also name one element of A.

    (b) Is B the empty set? If *yes*, just write '$B = \emptyset$'. If *no*, write '$B \neq \emptyset$' and also name one element of B.

13. Consider each of the statements below. Determine whether it is true or not. Justify your answer.

    You may take for granted that $\sqrt{5}$ is an irrational number whose value is between 2 and 3.

(a) $\sqrt{5} \in \{x \in \mathbb{R} : 1 \leq x < 3\}$.

(b) $\sqrt{5} \in \{x \in \mathbb{R} : 1 \leq x < 2\}$.

(c) $\sqrt{5} \in \{x \in \mathbb{Q} : 1 \leq x < 3\}$.

(d) $\sqrt{5} \in \{x \in \mathbb{R} : x^2 \in \mathbb{N}\}$.

(e) $\sqrt{5} \in \{x \in \mathbb{R} : x = -r^2 \text{ for some } r \in \mathbb{R}\}$.

(f) $\sqrt{5} \in \{x \in \mathbb{R} : x = a + b\sqrt{5} \text{ for some } a, b \in \mathbb{Z}\}$.

14. Consider each of the 'infinite' collections of objects below. Apply the Method of Specification to express the collection as a set.

(a) $$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \cdots, \frac{1}{2^n}, \frac{1}{2^{n+1}}, \cdots.$$

(b) $$1, \frac{3}{5}, \frac{9}{25}, \frac{27}{125}, \frac{81}{625}, \cdots, \left(\frac{3}{5}\right)^n, \left(\frac{3}{5}\right)^{n+1}, \cdots.$$

(c)

$$\begin{array}{ccccccccc}
1 & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} & \frac{1}{81} & \cdots & \frac{1}{3^n} & \frac{1}{3^{n+1}} & \cdots \\
2 & \frac{2}{3} & \frac{2}{9} & \frac{2}{27} & \frac{2}{81} & \cdots & \frac{2}{3^n} & \frac{2}{3^{n+1}} & \cdots \\
4 & \frac{4}{3} & \frac{4}{9} & \frac{4}{27} & \frac{4}{81} & \cdots & \frac{4}{3^n} & \frac{4}{3^{n+1}} & \cdots \\
8 & \frac{8}{3} & \frac{8}{9} & \frac{8}{27} & \frac{8}{81} & \cdots & \frac{8}{3^n} & \frac{8}{3^{n+1}} & \cdots \\
16 & \frac{16}{3} & \frac{16}{9} & \frac{16}{27} & \frac{16}{81} & \cdots & \frac{16}{3^n} & \frac{16}{3^{n+1}} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \\
2^m & \frac{2^m}{3} & \frac{2^m}{9} & \frac{2^m}{27} & \frac{2^m}{81} & \cdots & \frac{2^m}{3^n} & \frac{2^m}{3^{n+1}} & \cdots \\
2^{m+1} & \frac{2^{m+1}}{3} & \frac{2^{m+1}}{9} & \frac{2^{m+1}}{27} & \frac{2^{m+1}}{81} & \cdots & \frac{2^{m+1}}{3^n} & \frac{2^{m+1}}{3^{n+1}} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots &
\end{array}$$

(d)

$$\begin{array}{lllllll}
0 & 1-e & 1-e^2 & \cdots & 1-e^n & 1-e^{n+1} & \cdots \\
\pi-1 & \pi-e & \pi-e^2 & \cdots & \pi-e^n & \pi-e^{n+1} & \cdots \\
\pi^2-1 & \pi^2-e & \pi^2-e^2 & \cdots & \pi^2-e^n & \pi^2-e^{n+1} & \cdots \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \\
\pi^m-1 & \pi^m-e & \pi^m-e^2 & \cdots & \pi^m-e^n & \pi^m-e^{n+1} & \cdots \\
\pi^{m+1}-1 & \pi^{m+1}-e & \pi^{m+1}-e^2 & \cdots & \pi^{m+1}-e^n & \pi^{m+1}-e^{n+1} & \cdots \\
\vdots & \vdots & \vdots & & \vdots & \vdots &
\end{array}$$