1. **Statements**.

A **(mathematical) statement** is a sentence, or a number of carefully worded inter-related sentences, (with mathematical content,) for which it is meaningful to say it is true or it is false.

All statements are placed on equal footing:

* No prejudice towards any statement, whether true or false.
  (Example: '$1 + 1 = 2$', '$1 + 1 = 3$' are on equal footing as statements.)

Aristotle's Law of the Excluded Middle:

- each statement is true or false, but not both.

Truth values: T, F.

* A statement known to be true is assigned T (for 'truth').
* A statement known to be false is assigned F (for 'falsity').

2. **Decomposition of statements into blocks.**

Delete words indicating 'logical relations' in any given statement, such as

'and', 'or', 'if', 'then', 'suppose', 'assume', 'let',

and we will obtain 'simpler blocks' which are also statements.

Examples.

(a) ~~Let~~ ΔABC be a triangle. ~~Suppose~~ $\angle ACB$ is a right angle. ~~Then~~ $AB^2 = AC^2 + BC^2$.

(1) ΔABC is a triangle. (2) $\angle ACB$ is a right angle. (3) $AB^2 = AC^2 + BC^2$.

(b) Let $x, y \in \mathbb{Z}$. Suppose x is divisible by y and y is divisible by x. Then $|x| = |y|$.

(1) $x \in \mathbb{Z}$. (3) x is divisible by y. (5) $|x| = |y|$.

(2) $y \in \mathbb{Z}$. (4) y is divisible by x.

(c) Let A, B be sets. $A \subset B \backslash (B \backslash A)$ iff $A \subset B$.

(1) A is a set. (3) $A \subset B \backslash (B \backslash A)$.

(2) B is a set. (4) $A \subset B$.

This process can be reversed:

- $*$ Join statements with words indicating 'logical relations', such as

  'and', 'or', 'if', 'then', 'suppose', 'assume', 'let',

  and we will obtain more complicated statements.

Hence we have these notions:

- $*$ **Logical Connectives**:

  'and', 'or', 'if', 'then', 'suppose', 'assume' *et cetera*

- $*$ **Compound statements** are:
  - $\star$ statements obtained by joining several 'shorter' statements with logical connectives.

3. **Negation of a statement.**

Given a statement P, we may form these statements:

* ‘the statement P is true’,
* ‘the statement P is not true’.

Regarded to be the same:

$$P, \qquad \text{‘the statement } P \text{ is true’}.$$

$1+1=2$

‘$1+1=2$’ is true.

Regarded to be the same.

Reason: they are either true together, or false together.

‘$1+1=2$’ is false.

Opposite to each of:

- ‘$1+1=2$’
- ‘"$1+1=2$" is true.’

How about ‘the statement P is not true’?

* True exactly when P is false
* False exactly when P is true.

It is called the **negation** of P.

Notation: $\sim P$. Alternative notation: $\neg P$

Pronounciation: ‘not P’.

$1=2$.

‘$1=2$’ is true.

Regarded to be the same.

‘$1=2$’ is false.

Opposite to ‘$1=2$’.

Examples.

(a)

P	'$1+1=2$'	true statement
'P is true'	'$1+1=2$ is true'	true statement
$\sim P$	'$1+1=2$ is not true'	false statement

(b)

Q	'$1=2$'	false statement
'Q is true'	'$1=2$ is true'	false statement
$\sim Q$	'$1=2$ is not true'	true statement

The relation between P and $\sim P$, summarized in a table:

P	$\sim P$
T	F
F	T

4. **Conjunctions and disjunctions of two statements.**

Given two statements P, Q, we may form the compound statements using 'and', 'or':

$$\text{`}P \text{ and } Q\text{'}, \qquad \text{`}P \text{ or } Q\text{'}.$$

P: '1+1=2'.
Q: '3+2=7'
P and Q: '1+1=2 and 3+2=7'
P or Q: '1+1=2 or 3+2=7'

Conjunction of P, Q:

'P and Q'

Notation: $P \wedge Q$.

Pronunciation: 'P wedge Q', 'P and Q'

Truth values?

$*$ $P \wedge Q$ is true exactly when both P, Q are true.

The relation amongst P, Q, $P \wedge Q$ is summarized in a table:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction of P, Q:

'P or Q'

Notation: $P \vee Q$.

Pronunciation: 'P vee Q', 'P or Q'

Truth values?

* $P \vee Q$ is true exactly when at least one of P, Q is true.

The relation amongst P, Q, $P \vee Q$ is summarized in a table:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Beware! This 'or' here is different from that 'or' in daily language.

5. **Truth table, and logical equivalence.**

A table such as the one relating $P, Q, P \wedge Q$ with $(P \wedge Q) \vee Q$, $P \wedge [(P \wedge Q) \vee Q]$

P	Q	$P \wedge Q$	$(P \wedge Q) \vee Q$	$P \wedge [(P \wedge Q) \vee Q]$
T	T	T	T	T
T	F	F	F	F
F	T	F	T	F
F	F	F	F	F

is called a **truth table**.

How to read a truth table?

* Read row-by-row.
* 'Simpler blocks': put on the left-hand-side (in this example P, Q)
* 'Simplest ones' on the extreme left.
* Compound statements (in this example $P \wedge Q$): put on the right-hand-side.
* Each new column corresponds to a new compound statement formed by joining 'simpler ones' on its left by a 'logical connective'.

Examples.

(a) Truth table for $\sim(P \vee Q)$.

P	Q	$P \vee Q$	$\sim(P \vee Q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(b) Truth table for $(\sim P) \wedge (\sim Q)$.

P	Q	$\sim P$	$\sim Q$	$(\sim P) \wedge (\sim Q)$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(c) Truth table displaying $\sim(P \vee Q)$, $(\sim P) \wedge (\sim Q)$ 'simultaneously'.

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$\sim Q$	$(\sim P) \wedge (\sim Q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

$\sim(P \vee Q)$, $(\sim P) \wedge (\sim Q)$ are simultaneously true or simultaneously false, irrespective of the truth values of P, Q.

Hence we say that $\sim(P \vee Q)$, $(\sim P) \wedge (\sim Q)$ are logically equivalent.

(d) Truth table displaying the truth values of $\sim(P \vee Q)$, $(\sim P) \vee Q$ 'simultaneously'.

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$(\sim P) \vee Q$
T	T	T	F	F	T
T	F	T	F	F	F
F	T	T	F	T	T
F	F	F	T	T	T

$\sim(P \vee Q)$, $(\sim P) \vee Q$ are not logically equivalent. (Why?)

Note:
Brackets are important.

(e) Truth table displaying the truth values of $\sim(P \wedge Q)$, $(\sim P) \vee (\sim Q)$ 'simultaneously'.

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$(\sim P) \vee (\sim Q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$\sim(P \wedge Q)$, $(\sim P) \vee (\sim Q)$ are logically equivalent.

Remarks.

(1) Refer to (c).

$\sim(P \vee Q)$, $(\sim P) \wedge (\sim Q)$ are both true or both false, (irrespective of the respective truth values of P, Q).

For this reason, these two statements are the same as each other.

Hence we say they are **logically equivalent** to each other.

(2) Refer to (d).

$\sim(P \vee Q)$, $(\sim P) \vee Q$ are not logically equivalent.

Beware! Brackets matter.

'Not $x > 2$ or $x = 3$' is unclear.
Does it mean '(Not $x > 2$) or $x = 3$' ?
Or does it mean 'Not ($x > 2$ or $x = 3$)' ?

(3) Refer to (e). $\sim(P \wedge Q)$, $(\sim P) \vee (\sim Q)$ are logically equivalent to each other.

De Morgan's Laws in logic: the logical equivalence in (c), (e).

(4) Brackets indicate how a statement is supposed to be read.

Refer to (d). Does the chain of symbols '$\sim P \vee Q$' mean

'$\sim(P \vee Q)$' or '$(\sim P) \vee Q$'?

How about the chain of symbols '$\sim P \wedge Q$'?

Examples (Continued).

(f) $(P \wedge Q) \vee R$, $(P \vee R) \wedge (Q \vee R)$ are logically equivalent:

P	Q	R	$P \wedge Q$	$(P \wedge Q) \vee R$	$P \vee R$	$Q \vee R$	$(P \vee R) \wedge (Q \vee R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	T	F	F
F	T	T	F	T	T	T	T
F	T	F	F	F	F	T	F
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	F

(g) $(P \vee Q) \wedge R$, $(P \wedge R) \vee (Q \wedge R)$ are logically equivalent:

P	Q	R	$P \vee Q$	$(P \vee Q) \wedge R$	$P \wedge R$	$Q \wedge R$	$(P \wedge R) \vee (Q \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	T	T	F	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Remark.

(5) logical equivalence in (f), (g):
Distributive Laws for conjunction and disjunction in logic.

Examples (Continued).

(h) $(P \wedge Q) \vee R$, $P \wedge (Q \vee R)$ are not logically equivalent:

P	Q	R	$P \wedge Q$	$(P \wedge Q) \vee R$	$Q \vee R$	$P \wedge (Q \vee R)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	T
T	F	F	F	F	F	F
F	T	T	F	T	T	F
F	T	F	F	F	T	F
F	F	T	F	T	T	F
F	F	F	F	F	F	F

Remark.

(6) Note that $(P \wedge Q) \vee R$, $P \wedge (Q \vee R)$ are not logically equivalent.

Again beware! Brackets matter.

'$P \wedge Q \vee R$' does not make sense.

Example.

* Compare

'($x > 2$ and $x < 4$) or $x > 1$', 'x > 2 and ($x < 4$ or $x > 1$)'.

So '$x>2$ and $x<4$ or $x>1$' is unclear.

Examples of pairs of logically equivalent statements.

- **De Morgan's Laws**:

  $\sim(P \vee Q)$, $(\sim P) \wedge (\sim Q)$ are logically equivalent.
  $\sim(P \wedge Q)$, $(\sim P) \vee (\sim Q)$ are logically equivalent.

- **Distributive Laws for conjunction and disjunction**:

  $(P \vee Q) \wedge R$, $(P \wedge R) \vee (Q \wedge R)$ are logically equivalent.
  $(P \wedge Q) \vee R$, $(P \vee R) \wedge (Q \vee R)$ are logically equivalent.

- **Law of Double Negative Elimination**:

  P, $\sim(\sim P)$ are logically equivalent.

- **Law of Commutativity of Conjunction**:

  $P \wedge Q$, $Q \wedge P$ are logically equivalent.

- **Law of Commutativity of Disjunction**:

  $P \vee Q$, $Q \vee P$ are logically equivalent.

- **Law of Associativity of Conjunction**:

  $(P \wedge Q) \wedge R$, $P \wedge (Q \wedge R)$ are logically equivalent.

- **Law of Associativity of Disjunction**:

  $(P \vee Q) \vee R$, $P \vee (Q \vee R)$ are logically equivalent.

6. **Conditionals.**

Given two statements P, Q, we may form the compound statement using 'if', 'then' simultaneously:

$$\text{`if } P \text{ then } Q\text{'.}$$

This is the **conditional** from the **assumption** P to the **conclusion** Q.

Notation: $P \to Q$

Pronounciation: 'P arrow Q', 'if P then Q'

Truth values?

* $P \to Q$ is true except when P is true and Q is false.

The relation amongst $P, Q, P \to Q$, summarized in a truth table:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Whenever P is false, P→Q is true.

Various 'wordy' formulations for $P \to Q$:

(a) 'P only if Q'.

(b) 'Suppose P. Then Q.'

(c) 'Q is necessary for P'. 'Q is a **necessary condition** for P'.

(d) 'P is sufficient for Q'. 'P is a **sufficient condition** for Q'.

(e) 'Assuming/Given/Provided that P is true, Q is true'.

7. **Another way to see what conditionals are.**

Consider the statements

* $P \to Q$,
* $(\sim P) \vee Q$,

and the latter's 'double negation'

* $\sim[P \wedge (\sim Q)]$.

Truth table showing truth values of $P \to Q$, $(\sim P) \vee Q$, $\sim[P \wedge (\sim Q)]$ simultaneously:

P	Q	$P \to Q$	$\sim P$	$\sim Q$	$(\sim P) \vee Q$	$P \wedge (\sim Q)$	$\sim[P \wedge (\sim Q)]$
T	T	T	F	F	T	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	T	F	T
F	F	T	T	T	T	F	T

'$P \to Q$', '$(\sim P) \vee Q$', '$\sim[P \wedge (\sim Q)]$' are logically equivalent.

The logical equivalence between '$P \to Q$' and '$\sim[P \wedge (\sim Q)]$' is the logical foundation of the 'proof-by-contradiction' method.

8. **Converse, contrapostive and inverse of a conditional.**

Consider the statements $P, Q, P \to Q$.

- The statement $Q \to P$ is called the **converse** of the conditional $P \to Q$.
- The statement $(\sim Q) \to (\sim P)$ is called the **contrapositive** of the conditional $P \to Q$.
- The statement $(\sim P) \to (\sim Q)$ is called the **inverse** of the conditional $P \to Q$.

Respective truth values?

logically equivalent.

P	Q	$P \to Q$	$Q \to P$	$\sim P$	$\sim Q$	$(\sim P) \to (\sim Q)$	$(\sim Q) \to (\sim P)$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

not logically equivalent.

not logically equivalent.

Therefore,

- $P \to Q$, $(\sim Q) \to (\sim P)$ are logically equivalent,
- $P \to Q$, $Q \to P$ are not logically equivalent,
- $P \to Q$, $(\sim P) \to (\sim Q)$ are not logically equivalently, and
- $Q \to P$, $(\sim P) \to (\sim Q)$ are logically equivalent.

To justify one of $P \to Q$, $(\sim Q) \to (\sim P)$ is the same as to justify the other.

This is the logical foundation of the 'contrapositive proof'.

A statement and its converse are not logically equivalent.

* $*$ It can happen that both are true.
* $*$ It can happen that both are false.
* $*$ It can also happen that one is true while the other is false.

Examples.

(a) P: 'the quadrilateral $ABCD$ is a square.'
Q: 'all angles of the quadrilateral $ABCD$ are right angles.'
$P \to Q$ is true. $Q \to P$ is false.

(b) P: '$\triangle ABC$ is equilateral.'
Q: 'all three angles in $\triangle ABC$ are equal to each other.'
$P \to Q$ is true. $Q \to P$ is true.

(c) P: '$\triangle ABC$ is an isosceles triangle'.
Q: '$\triangle ABC$ is a right-angle triangle'.
$P \to Q$ is false. $Q \to P$ is false.

In Euclidean geometry, there are a lot of pairs of conditionals and converses which are both true. Examples:

(a) Parallel Postulate and its converse (Fifth Postulate, and Proposition 27 of Book I, *Euclid's Elements*).

(b) Pythagoras' Theorem and its converse (Propositions 47, 48 of Book I, *Euclid's Elements*).

(c) Thales' Theorem (Proposition 31 of Book III, *Euclid's Elements*) and its converse.

In your *analysis* course, you will find a lot of conditionals which are true but whose respective converses are false. These are the simplest examples:

(a) (...) Suppose f is differentiable at c. Then f is continuous at c.

(b) (...) Suppose f is continuous on $[a, b]$. Then f is integrable on $[a, b]$.

9. **Biconditionals.**

Given two statements P, Q, we may form the compound statement

$$\text{‘}P \text{ if and only if } Q\text{’},$$

using the phrase ‘if and only if’.

Short-hand: ‘P iff Q’.

This is called the **biconditional** from P to Q.

Notation: $P \leftrightarrow Q$.

Pronouciation: ‘P double-arrow Q’, ‘P if and only if Q’.

Truth values?

* The statement $P \leftrightarrow Q$ is true exactly when P, Q are both true or both false.

Truth table:

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Various ‘wordy’ formulations for $P \leftrightarrow Q$:

(a) ‘P is necessary and sufficient for Q’.

(b) ‘P is a **necessary and sufficient condition** for Q’.

(c) ‘Q is a necessary and sufficient condition for P’.

(d) ‘P, Q are **(logically) equivalent** to each other’.

$P \leftrightarrow Q$, $(P \to Q) \wedge (Q \to P)$ are logically equivalent:

P	Q	$P \leftrightarrow Q$	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

So these are the same:

$*$ ‘$P \leftrightarrow Q$ is true’

$*$ ‘$(P \to Q) \wedge (Q \to P)$ is true’,

$*$ both of ‘$P \to Q$ is true’,‘$Q \to P$ is true’.

To justify one of them is the same as to justify the other.

10. **Tautologies, contradictions and contingent statements.**

Consider a compound statement

$$\Sigma(P, Q, R, \cdots)$$

formed by 'connecting' a number of statements $P, Q, R, \cdots$ with the 'logical connectives'

$$\sim, \vee, \wedge, \rightarrow, \leftrightarrow.$$

- The statement $\Sigma(P, Q, R, \cdots)$ is called a **tautology** exactly when:

  it is always true irrespective of the truth values of $P, Q, R, \cdots$.

Examples:

(a) $P \vee (\sim P)$.

(b) $P \rightarrow P$.

(c) $P \leftrightarrow [\sim(\sim P)]$.

(d) $(P \wedge Q) \rightarrow P$.

(e) $P \rightarrow (P \vee Q)$.

(f) $[P \wedge (Q \vee R)] \leftrightarrow [(P \wedge Q) \vee (P \wedge R)]$.

P	$\sim P$	$P \vee (\sim P)$
T	F	T
F	T	T

P	Q	$P \wedge Q$	$(P \wedge Q) \rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

- The statement $\Sigma(P, Q, R, \cdots)$ is called a **contradiction** exactly when:

  it is always false irrespective of the truth values of $P, Q, R, \cdots$.

Examples:

(a) $P \wedge (\sim P)$.

(b) $P \leftrightarrow (\sim P)$.

P	$\sim P$	$P \wedge (\sim P)$
T	F	F
F	T	F

- The statement $\Sigma(P, Q, R, \cdots)$ is called a **contingent statement** exactly when:

  it is neither a tautology nor a contradiction.

Examples:

(a) P.

(b) $P \wedge Q$.

(c) $P \vee Q$.

(d) $P \rightarrow Q$.

(e) $(\sim P) \rightarrow P$.

(f) $P \rightarrow (P \wedge Q)$.

(g) $(P \vee Q) \rightarrow P$.

P	$\sim P$	$(\sim P) \rightarrow P$
T	F	F
F	T	T

P	Q	$P \wedge Q$	$P \rightarrow (P \wedge Q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

11. **Another view on logical equivalence**.

Given statements $P, Q, \cdots$, we form two compound statements

$$\Sigma(P, Q, \cdots), \qquad \Sigma'(P, Q, \cdots),$$

and then further form the compound statement

$$\Sigma(P, Q, \cdots) \leftrightarrow \Sigma'(P, Q, \cdots).$$

In general such a statement needs not be a tautology.

It is a tautology exactly when:

$$\Sigma(P, Q, \cdots), \Sigma'(P, Q, \cdots) \text{ are logically equivalent.}$$

Examples of logical equivalence.

(a) Law of Double Negation: $P \leftrightarrow [\sim(\sim P)]$.

(b) Distributive Law:
$[(P \wedge Q) \vee R] \leftrightarrow [(P \vee R) \wedge (Q \vee R)]$, $[(P \vee Q) \wedge R] \leftrightarrow [(P \wedge R) \vee (Q \wedge R)]$.

(c) De Morgan's Law: $[\sim(P \wedge Q)] \leftrightarrow [(\sim P) \vee (\sim Q)]$, $[\sim(P \vee Q)] \leftrightarrow [(\sim P) \wedge (\sim Q)]$.

(d) $(P \rightarrow Q) \leftrightarrow [(\sim P) \vee Q]$.

(e) $(P \rightarrow Q) \leftrightarrow \{\sim[P \wedge (\sim Q)]\}$.

(f) $(P \rightarrow Q) \leftrightarrow [(\sim Q) \rightarrow (\sim P)]$.

12. **Rules of inference**.

Given statements $P, Q, \cdots$, we form compound statements

$$\Sigma_1(P, \cdots),\ ...,\ \Sigma_n(P, \cdots),\ \Sigma'(P, \cdots),$$

and then further form the compound statement

$$(\Sigma_1(P, \cdots) \wedge \cdots \wedge \Sigma_n(P, \cdots)) \rightarrow \Sigma'(P, \cdots).$$

In general such a statement needs not be a tautology.

It is called a **rule of inference** exactly when it is a tautology. It is usually presented in the table form

$$\begin{array}{c} \Sigma_1(P, Q, \cdots) \\ \Sigma_2(P, Q, \cdots) \\ \vdots \\ \Sigma_n(P, Q, \cdots) \\ \hline \Sigma'(P, Q, \cdots) \end{array}$$

Examples of rules of inference.

(a) **Modus Ponens**: $[(P \to Q) \wedge P] \to Q$.

(b) **Modus Tollendo Ponens**: $[(P \vee Q) \wedge (\sim P)] \to Q$, $[(P \vee Q) \wedge (\sim Q)] \to P$.

(c) **Modus Tollens**: $[(P \to Q) \wedge (\sim Q)] \to (\sim P)$.

(d) **Hypothetical syllogism**: $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$.

(e) **Biconditional-conditional**: $(P \leftrightarrow Q) \to (P \to Q)$, $(P \leftrightarrow Q) \to (Q \to P)$.

(f) **Conditional-biconditional**: $[(P \to Q) \wedge (Q \to P)] \to (P \leftrightarrow Q)$.

(g) **Simplification**: $(P \wedge Q) \to P$, $(P \wedge Q) \to Q$.

(h) **Addition**: $P \to (P \vee Q)$, $Q \to (P \vee Q)$.

(i) **Repetition**: $P \to P$.

(j) **Double negation**: $[\sim(\sim P)] \to P$.

(k) **Adjunction**: $(P \wedge Q) \to (P \wedge Q)$.

(l) **Constructive dilemma**: $[(P \to Q) \wedge (R \to S) \wedge (P \vee R)] \to (Q \vee S)$.

(m) **Idempotency of entailment**: $[P \to (P \to Q)] \to (P \to Q)$.

(n) **Monotonicity of entailment**: $(P \to Q) \to [(P \wedge R) \to Q]$.

Modus Ponens and the symbol '$\Rightarrow$'.

P	Q	$P \to Q$	$(P \to Q) \wedge P$	$[(P \to Q) \wedge P] \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

So $[(P \to Q) \wedge P] \to Q$ is indeed a tautology. This rule of inference is known as Modus Ponens.

Some mathematicians choose to indicate the use of Modus Ponens with the help of the symbol '$\Rightarrow$':

- What is meant by '$G \Rightarrow H$'? What does it stand for?

  Answer. The statement G is (known/asserted to be) true.
  The conditional '$G \to H$' is also (known to be) true.
  So by Modus Ponens, it may be concluded that H is true.

- What is meant by '$G \Rightarrow H \Rightarrow J$'? What does it stand for?

  Answer. The statement G is (known/asserted to be) true.
  The conditionals '$G \to H$', '$H \to J$' are also (known to be) true.
  So by two successive applications of Modus Ponens, it may be concluded that J is true.

Mechanism behind proof-by-contradiction method

Given : Statements H, K.

Try to prove (☆): 'Suppose H is true. Then K is true.'

Method : 'Suppose $H \wedge (\sim K)$ is true.

Deduce an appropriate ~~statement~~ C which is known to be a contradiction.

Hence conclude that (☆) holds in the first place.'

Mechanism? Ask: What are '?', '??', '???' in the excerpt of ~~the~~ the truth table below?

H	K	$\sim K$	$H \wedge (\sim K)$	C	$[H \wedge (\sim K)] \to C$
T	???	??	?	F	T