MATH1050 Answers to Examples: Complex numbers.

1. (a) $\omega^2 = i$, $\omega^8 = 1$, $\omega^{2016} = \omega^{252\cdot 8} = 1$.

   (b) $\left|\sum_{k=0}^{2017} \omega^k\right|^2 = 2 + \sqrt{2}$.

2. $\zeta + \bar{\zeta} = 2\mathsf{Re}(\zeta) = 2a$.

   $\zeta^2 + \bar{\zeta}^2 = 4a^2 - 2r^2$.

   $\zeta^3 + \bar{\zeta}^3 = 8a^3 - 6ar^2$.

   $\zeta^4 + \bar{\zeta}^4 = 16a^4 - 16a^2r^2 + 2r^4$.

   $\zeta^5 + \bar{\zeta}^5 = 32a^5 - 40a^3r^2 + 10ar^4$.

   $\zeta^6 + \bar{\zeta}^6 = 64a^6 - 96a^4r^2 + 36a^2r^4 - 2r^6$.

   Remark. One possible approach is to make good use of binomial expansions.

3. (a) $\mathsf{Re}(\zeta) = 2k^2 - 3k - 2$ and $\mathsf{Im}(\zeta) = k^2 - 3k + 2$.

   i. One possibility is $k = 1$ and $\zeta = -3$.
   The other is $k = 2$ and $\zeta = 0$.

   ii. One possibility is $k = -\frac{1}{2}$ and $\zeta = \frac{15}{4}i$. The other is $k = 2$ and $\zeta = 0$.

   iii. $k = -2$ and $\zeta = 12 + 12i$.

4. (a) ——

   (b) ——

5. (a) ——

   (b) $z = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) w$ or $z = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) w$.

6. (a) 1

   (b) ——

7. (a) $\lambda\mu\nu = 1$.

   (b) ——

   (c) ——

8. ——

9. $a = -\frac{7}{2}$, $b = \frac{1}{2}$.

10. $a = -2$ and $b = 2$.

11. ——

12. (a) ——

    (b) $z = 0$ or $z = 1$ or $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ or $z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$.

13. The solutions of the system $(\star)$ is given by $z = 1 + \sqrt{3}i$ or $z = 1 - \sqrt{3}i$.

14. $(2 + 2.5\sqrt{2}) + (-3 - 2.5\sqrt{2})i$, $(2 - 2.5\sqrt{2}) + (-3 + 2.5\sqrt{2})i$.

15. $2 + 3i$.

16. (a) $\mathsf{Im}(z) = -\frac{1}{2}\mathsf{Re}(z) + 5$.

(b) $2\sqrt{5}$.

17. $-1-i=\sqrt{2}\Big(\cos(-\frac{3\pi}{4})+i\sin(-\frac{3\pi}{4})\Big)$.

$1-i=\sqrt{2}\Big(\cos(-\frac{\pi}{4})+i\sin(-\frac{\pi}{4})\Big)$.

$\frac{-1-i}{(1-i)^5}=\frac{i}{4}$

18. -3.

19. (a) $p=\sqrt{3}+i$.

$r=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$.

(b) $q=\frac{2\sqrt{3}-1}{2}+\frac{2+\sqrt{3}}{2}i$.

20. (a) ——

(b) i. $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i,\ -\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$.

ii. $\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i,\ -\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$.

iii. $\frac{1}{2}+\frac{\sqrt{3}}{2}i,\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$.

iv. $\frac{\sqrt{3}}{2}+\frac{\sqrt{1}}{2}i,\ -\frac{\sqrt{3}}{2}-\frac{\sqrt{1}}{2}i$.

v. $\frac{\sqrt{\sqrt{10}+1}}{\sqrt{2}}+\frac{\sqrt{\sqrt{10}-1}}{\sqrt{2}}i,\ -\frac{\sqrt{\sqrt{10}+1}}{\sqrt{2}}-\frac{\sqrt{\sqrt{10}-1}}{\sqrt{2}}i$.

vi. $\frac{\sqrt{\sqrt{2}+1}}{\sqrt{2}}+\frac{\sqrt{\sqrt{2}-1}}{\sqrt{2}}i,\ -\frac{\sqrt{\sqrt{2}+1}}{\sqrt{2}}-\frac{\sqrt{\sqrt{2}-1}}{\sqrt{2}}i$.

21. (a) $z=0$ or $z=-4i$.

(b) $z=-3$ or $z=2i$.

(c) $z=(1+\sqrt{2})i$ or $z=(1-\sqrt{2})i$.

(d) $z=2+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$ or $z=2+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$.

(e) $z=3+i$ or $z=-1+i$.

(f) $z=2i$ or $z=3i$.

22. (a) i. $\frac{3\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,\ -\frac{3\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$.

ii. $\frac{3\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i,\ -\frac{3\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$.

(b) $x=\frac{3\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ or $x=-\frac{3\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ or $x=\frac{3\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ or $x=-\frac{3\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$.