

MATH1050 Answers to Examples: Binomial coefficients and Binomial Theorem.

1. (a) $k_1 = 6n$, $k_2 = 6an + 3n(6n - 1)$, $k_3 = 6an(6n - 1) + n(6n - 1)(6n - 2)$.

(b) i. $A = -18$, $B = 27$, $C = -7$.

ii. $n = 1$.

2. (a) $A = 1$, $B = 3$, $C = 2$.

(b) $D = 2$, $E = 1$, $F = 1$.

3. $A = 9$, $B = 1$.

4. —

5. (a) $\sum_{j=0}^n \binom{n}{j}^2 = \binom{2n}{n}$.

Remark. Make use of the equality $(1+x)^{2n} = (1+x)^n(1+x)^n$ as polynomials. Express the coefficient of x^n in two different ways, one according to one side of this equality.

(b) $\sum_{j=0}^n (-1)^j \binom{n}{j}^2 = \begin{cases} (-1)^{n/2} \binom{n}{n/2} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$.

Remark. Make use of the equality $(1-x^2)^n = (1+x)^n(1-x)^n$ as polynomials. Express the coefficient of x^n in two different ways, one according to one side of this equality.

6. (a) $A = 2$, $B = 4$, $C = 3$.

(b) $A = 3$, $B = 3$, $C = 8$.

7. (a) i. —

ii. A. $\binom{n+m+1}{k+1} - \binom{n}{k+1}$.

B. $\binom{n+m+1}{k+1}$.

(b) i. —

ii. $24 \binom{m+5}{5}$

8. —