MATH1050 Answers to Examples: Equations involving trigonometric functions.

1. (a) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + K \cdot \pi \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = (-1)^M \cdot (-\frac{\pi}{6}) + M\pi \text{ for some } M \in \mathbb{Z}\right\}. \end{aligned}$$

The solution set of the equation $\sin(2x) + \cos(x) = 0$ is given by $A \cup B$.

(b) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{\pi}{4} + K \cdot \pi \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = -\frac{\pi}{4} + M\pi \text{ for some } M \in \mathbb{Z}\right\}. \end{aligned}$$

The solution set of the equation $\tan^2(x) + 3 = 2\sec^2(x)$ is given by $A \cup B$.

(c) The solution set of the equation $\cos(3x) = \cos(x)$ is given by $\left\{x \in \mathbb{R} : x = \frac{K \cdot \pi}{2} \text{ for some } K \in \mathbb{Z}\right\}$

(d) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{K \cdot \pi}{2} \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{3} + 2M\pi \text{ for some } M \in \mathbb{Z}\right\}, \\ C &= \left\{x \in \mathbb{R} : x = -\frac{\pi}{3} + 2M\pi \text{ for some } M \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation $\sin(x) + \sin(2x) + \sin(3x) = 0$ is given by $A \cup B \cup C$.

(e) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = (-1)^K \cdot \frac{\pi}{18} + K \cdot \frac{\pi}{3} \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + M\pi \text{ for some } M \in \mathbb{Z}\right\}. \end{aligned}$$

The solution set of the equation $\sin(2x) + \sin(4x) = \cos(x)$ is given by $A \cup B$.

(f) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{\pi}{9} + K \cdot \frac{2\pi}{3} \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = -\frac{\pi}{9} + L \cdot \frac{2\pi}{3} \text{ for some } L \in \mathbb{Z}\right\}, \\ C &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + M\pi \text{ for some } M \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation $\cos(4x) + \cos(2x) = \cos(x)$ is given by $A \cup B \cup C$.

(g) Denote by α the number given by $\alpha = \arcsin(\frac{1}{4})$.

Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = (-1)^K \alpha + K \cdot \pi \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + M \cdot 2\pi \text{ for some } M \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation $2\cos(2x) + 5\sin(x) - 3 = 0$ is given by $A \cup B$.

(h) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = (-1)^K \cdot \frac{\pi}{24} + K \cdot \frac{\pi}{4} \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + M\pi \text{ for some } M \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation $\sin(5x) + \sin(3x) = \cos(x)$ is given by $A \cup B$.

(i) Denote by α the number given by $\alpha = \arcsin(\frac{5}{13})$.

The solution set of the equation $12\cos(3x) - 5\sin(3x) = 13$ is given by $\left\{x \in \mathbb{R} : x = -\frac{\alpha}{3} + N \cdot \frac{2\pi}{3} \text{ for some } N \in \mathbb{Z}\right\}$.

(j) Denote by α the number given by $\alpha = \arcsin(\frac{1}{4})$.

The solution set of the equation $\sin(3x + \frac{\pi}{4})\cos(3x - \frac{\pi}{4}) = \frac{3}{4}$ is given by

$\left\{x \in \mathbb{R} : x = -(-1)^N \cdot \frac{\alpha}{6} + N \cdot \frac{\pi}{6} \text{ for some } N \in \mathbb{Z}\right\}$.

(k) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{\pi}{9} + M \cdot \frac{2\pi}{3} \text{ for some } M \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + N \cdot \pi \text{ for some } N \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation $\cos(4x) - 2\sin^2(x) = -2\sin^2(\frac{x}{2})$ is given by $A \cup B$.

(l) The solution set of the equation $\sin(\frac{2}{x}) = \frac{1}{2}$ is given by $\left\{x \in \mathbb{R} : x = \frac{12}{6N + (-1)^N} \cdot \frac{1}{\pi} \text{ for some } N \in \mathbb{Z}\right\}$.

(m) The solution set of the equation $\cot(\frac{x^3}{3}) = -\sqrt{3}$ is given by $\left\{x \in \mathbb{R} : x = \sqrt[3]{\frac{\pi}{2} + N \cdot 3\pi} \text{ for some } N \in \mathbb{Z}\right\}$.

(n) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \sqrt{N} \cdot \frac{\sqrt{\pi}}{2} \text{ for some } N \in \mathbb{N}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = -\sqrt{N} \cdot \frac{\sqrt{\pi}}{2} \text{ for some } N \in \mathbb{N}\right\}. \end{aligned}$$

The solution set of the equation $\cos(4x^2) = -1$ is given by $A \cup B$.

(o) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{1}{K^2 \cdot \pi^2} \text{ for some } K \in \mathbb{N}\backslash\{0\}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \sqrt{M} \cdot \sqrt{\pi} \text{ for some } M \in \mathbb{N}\backslash\{0\}\right\} \\ C &= \left\{x \in \mathbb{R} : x = -\sqrt{M} \cdot \sqrt{\pi} \text{ for some } M \in \mathbb{N}\backslash\{0\}\right\}. \end{aligned}$$

The solution set of the equation $\sin(\frac{1}{\sqrt{x}})\sin(x^2) = 0$ is given by $A \cup B \cup C$.

(p) Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{1}{K \cdot \pi} \text{ for some } K \in \mathbb{Z}\backslash\{0\}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{3}{6M + 1} \cdot \frac{1}{\pi} \text{ for some } M \in \mathbb{Z}\right\}, \\ C &= \left\{x \in \mathbb{R} : x = \frac{3}{6M - 1} \cdot \frac{1}{\pi} \text{ for some } M \in \mathbb{Z}\right\}. \end{aligned}$$

The solution set of the equation $\sin(\frac{2}{x}) = \sin(\frac{1}{x})$ is given by $A \cup B \cup C$.

2. Let

$$\begin{aligned} A &= \left\{ x \in \mathbb{R} : x = \frac{\pi}{10} + K \cdot \frac{2\pi}{5} \text{ for some } K \in \mathbb{Z} \right\}, \\ B &= \{ x \in \mathbb{R} : x = M\pi \text{ where } M \in \mathbb{Z} \}. \end{aligned}$$

The solution set of the equation concerned is given by $A \cup B$.

3. Let

$$\begin{aligned} A &= \left\{ x \in \mathbb{R} : x = \frac{3\pi}{20} + K \cdot \frac{\pi}{5} \text{ for some } K \in \mathbb{Z} \right\} \\ B &= \left\{ x \in \mathbb{R} : x = \frac{\pi}{2} + M\pi \text{ for some } M \in \mathbb{Z} \right\} \end{aligned}$$

The solution set of the equation concerned is given by $A \cup B$.

4. Let

$$\begin{aligned} A &= \{ x \in \mathbb{R} : \ x = 2K\pi \text{ for some } K \in \mathbb{Z} \}, \\ B &= \left\{ x \in \mathbb{R} : x = \frac{\pi}{5} + L \cdot \frac{2\pi}{5} \text{ for some } L \in \mathbb{Z} \right\}, \\ C &= \left\{ x \in \mathbb{R} : x = \frac{\pi}{2} + M\pi \text{ for some } M \in \mathbb{Z} \right\}. \end{aligned}$$

The solution set of the equation concerned is given by $A \cup B \cup C$.

5. Let

$$\begin{aligned} A &= \left\{ x \in \mathbb{R} : x = \pm\frac{\pi}{9} + K \cdot \frac{2\pi}{3} \text{ for some } K \in \mathbb{Z} \right\}, \\ B &= \{ x \in \mathbb{R} : x = M\pi \text{ for some } M \in \mathbb{Z} \}. \end{aligned}$$

The solution set of the equation concerned is given by $A \cup B$.

6. (a) i. ——
   ii. ——

   (b) i. ——

   ii. $k \leq \frac{1}{3}$ or $k \geq 3$.

   (c) The solution set of the equation concerned is given by $\left\{ x \in \mathbb{R} :: x = \frac{\pi}{36} + (-1)^N \mu + N\pi \text{ for some } N \in \mathbb{Z} \right\}$.