MATH1050 Answers to Examples: Solving equations and inequalities with algebraic methods.

1. (a) The only solution of the equation $\dfrac{4x-7}{3x+5} = \dfrac{5}{3}$ is given by $x = -\dfrac{46}{3}$.

   (b) The only solution of the equation $\dfrac{x}{x-2} - \dfrac{x+1}{x-1} = \dfrac{x-8}{x-6} - \dfrac{x-9}{x-7}$ is given by $x = 4$.

   (c) The solutions of the equation $\dfrac{x^2-1}{x^2+1} = \dfrac{1}{2}$ are given by $x = -\sqrt{3}$ or $x = \sqrt{3}$.

   (d) The solutions of the equation $\dfrac{1}{x^3-x^2-x+1} + \dfrac{1}{x^3-3x^2-x+3} = \dfrac{2}{x^3-x^2-2x}$ are given by $x = -\dfrac{3}{2} + \dfrac{\sqrt{33}}{2}$ or $x = -\dfrac{3}{2} - \dfrac{\sqrt{33}}{2}$.

2. (a) The only solution of the equation $\sqrt{2x+9} = x - 3$ is given by $x = 8$.

   (b) There is no real solution for the equation $\sqrt{2x-3} = \sqrt{1-2x}$.

   (c) The only solution of the equation $\sqrt{x} - \dfrac{6}{\sqrt{x}} = 1$ is given by $x = 9$.

   (d) The only solution of the equation $\sqrt{x} - \sqrt{x-2} = 1$ is given by $x = \dfrac{9}{4}$.

   (e) The only solution of the equation $\sqrt{2x+3} + \sqrt{x+1} = \sqrt{8x+1}$ is given by $x = 3$.

   (f) The only solution of the equation $\sqrt{5x+1} + \sqrt{x+1} = \sqrt{10x+6}$ is given by $x = 3$.

   (g) The only solution of the equation $\sqrt{x^2+5x+2} = 1 + \sqrt{x^2+5}$ is given by $x = 2$.

   (h) The only solution of the equation $\dfrac{\sqrt{x}+9}{\sqrt{x}-6} = \dfrac{\sqrt{x}-5}{\sqrt{x}-13}$ is given by $x = 441$.

   (i) The solutions of the equation $\dfrac{1}{\sqrt{x^2-1}-x} + \dfrac{1}{\sqrt{x^2-1}+x} = -8$ are given by $x = -\sqrt{17}$ or $x = \sqrt{17}$

3. (a) The only solution of the equation $3^{2x+1} - 25 \cdot 3^x - 18 = 0$ is given by $x = 2$.

   (b) The solutions of the equation $5^{x+1} + 4 \cdot 5^{1-x} = 25$ are given by $x = 0$ or $x = \log_5(4)$.

   (c) The solutions of the equation $2^{(x^2-1)} \cdot 3^{2x-3} = 24$ are given by $x = 2$ or $x = -2 - 2\log_2(3)$.

   (d) The only solution of the equation $\ln(x) + \ln(2x-1) = 0$ is given by $x = 1$.

   (e) The solutions of the equation $\log_{10}(x^2+1) - \log_{10}(x-2) = 1$ are given by $x = 3$ or $x = 7$.

   (f) The solutions of the equation $\log_2(x) - \log_x(8) = 2$ are given by $x = \dfrac{1}{2}$ or $x = 8$.

   (g) The solution of the equation $\log_{10}(x^2+9) - 2\log_{10}(x) = 1$ is given by $x = 1$.

   (h) The only solution of the equation $\log_2(x+1) + \log_2(x+4) = 1 + 2\log_2(3)$ is given by $x = 2$.

   (i) The only solution of the equation $\log_3(\log_2(x)) + 2\log_9(\log_7(8)) = 2$ is given by $x = 343$.

   (j) The solutions of the equation $(\ln(x))^2 = \ln(x^2)$ are given by $x = e^2$ or $x = 1$.

   (k) The only solution of the equation $2\ln(x^{\ln(x)}) + 5\ln(x) = 3$ is given by $x = \sqrt{e}$ or $x = e^{-3}$.

4. (a) The solutions of the equation $|3x-5| = 31$ are given by $x = 12$ or $x = -\dfrac{26}{3}$.

   (b) The solutions of the equation $3|x-2| = 10$ are given by $x = -4/3$ or $x = 16/3$.

   (c) The solutions of the equation $|2 - 1/x| = 3$ are given by $x = -1$ or $x = 1/5$.

   (d) The solutions of the equation $|x^2 - 5x| = 6$ are given by $x = -1$ or $x = 6$ or $x = 2$ or $x = 3$.

   (e) The solutions of the equation $|x^2 + x - 13| = 7$ are given by $x = -5$ or $x = 4$ or $x = -3$ or $x = 2$.

   (f) The solutions of the equation $|x^2 - 5x + 2| = 2$ are given by $x = 0$ or $x = 1$ or $x = 4$ or $x = 5$.

   (g) The only solution of the equation $2x = |x-2|$ is given by $x = \dfrac{2}{3}$.

(h) The solutions of the equation $|x-1| = |x^2-4x+3|$ are given by $x = 1$ or $x = 2$ or $x = 4$.

(i) The solutions of the equation $|x-3| = |x^2-4x+3|$ are given by $x = 0$ or $x = 2$ or $x = 3$.

(j) The solutions of the equation $|x-1| = |x| - 1$ are given by $x \geq 1$.

(k) The solutions of the equation $|x^2-x-8| = |4x-2|$ are given by $x = -5$ or $x = -1$ or $x = 2$ or $x = 6$.

(l) The solutions of the equation $|x^2-4| = x-2$ are given by $x = -3$ or $x = -1$ or $x = 2$.

(m) The solutions of the equation $(x-3)^2 - |x-3| - 12 = 0$ are given by $x = -1$ or $x = 7$.

(n) The solutions of the equation $(x-5)^2 - 2|x-5| - 8 = 0$ are given by $x = 1$ or $x = 9$.

(o) The solutions of the equation $(x-1)|x| = x|x-1|$ are given by $x \leq 0$ or $x \geq 1$.

5. (a) The solutions of the equation $x = x$ are given by all real numbers.

(b) The solutions of the equation $0 \cdot x = 0$ are given by all real numbers.

(c) There is no solution for the equation $\dfrac{x^2-2x+1}{x-1} = 0$.

(d) There is no solution for the equation $\dfrac{x}{x-1} = \dfrac{1}{x-1}$.

(e) The only solution of the equation $\dfrac{x^2-1}{x-1} = 0$ is given by $x = -1$.

(f) The solutions of the equation $\dfrac{x}{x} = 1$ are given by all real numbers other than 0.

(g) The solutions of the equation $\dfrac{1}{x-1} = \dfrac{1}{x-1}$ are given by all real numbers other than 1.

(h) The solutions of the equation $\dfrac{1}{x-1} = \dfrac{x+1}{x^2-1}$ are given by all real numbers other than $1, -1$.

6. (a) The solutions of the system $\begin{cases} 3x + 2y = 5 \\ x^2 - 4xy + 3 = 0 \end{cases}$ are given by $(x,y) = \left(\dfrac{3}{7}, \dfrac{13}{7}\right)$ or $(x,y) = (1,1)$.

(b) The solutions of the system $\begin{cases} 3x^2 - xy - y^2 = 3 \\ x + y = 9 \end{cases}$ are given by $(x,y) = (4,5)$ or $(x,y) = (-7,16)$.

(c) The solutions of the system $\begin{cases} 2x^2 - y^2 = 2y \\ 6x^2 + xy - y^2 = 8y \end{cases}$ are given by $(x,y) = (0,0)$ or $(x,y) = (-2,2)$ or $(x,y) = \left(-\dfrac{6}{7}, \dfrac{4}{7}\right)$.

(d) The solutions of the system $\begin{cases} x^2 - xy - y^2 = y \\ x^2 - 4y^2 = 0 \end{cases}$ are given by $(x,y) = (0,0)$ or $(x,y) = (2,1)$ or $(x,y) = \left(-\dfrac{2}{5}, \dfrac{1}{5}\right)$.

(e) The solutions of the system $\begin{cases} 1/x^2 + 1/y^2 = 34 \\ 15xy = 1 \end{cases}$ are given by $(x,y) = \left(\dfrac{1}{5}, \dfrac{1}{3}\right)$ or $(x,y) = \left(-\dfrac{1}{5}, -\dfrac{1}{3}\right)$ or $(x,y) = \left(\dfrac{1}{3}, \dfrac{1}{5}\right)$ or $(x,y) = \left(-\dfrac{1}{3}, -\dfrac{1}{5}\right)$.

(f) The solutions of the system $\begin{cases} x^2 + y^2 = 5 \\ 1/x^2 + 1/y^2 = 5/4 \end{cases}$ are given by $(x,y) = (1,2)$ or $(x,y) = (-1,2)$ or $(x,y) = (-1,-2)$ or $(x,y) = (1,-2)$ or $(x,y) = (2,1)$ or $(x,y) = (-2,1)$ or $(x,y) = (-2,-1)$ or $(x,y) = (2,-1)$.

(g) The solutions of the system $\begin{cases} x/y + y/x = 17/4 \\ x^2 - 4xy + y^2 = 1 \end{cases}$ are given by $(x,y) = (1,4)$ or $(x,y) = (-1,-4)$ or $(x,y) = (4,1)$ or $(x,y) = (-4,-1)$.

(h) The only solution of the system $\begin{cases} x & - & y & = & 3 \\ \log_{10}(x) & + & \log_{10}(y) & = & 1 \end{cases}$ is given by $(x, y) = (5, 2)$.

7. (a) The solutions of the inequality $x^2 \geq 5x - 6$ are given by $x \leq 2$ or $x \geq 3$.

(b) The solutions of the inequality $(x-2)(x+3) < 2(x-2)$ are given by $-1 < x < 2$.

(c) The solutions of the inequality $(x+8)(2x-3) < (x-5)(x+8)$ are given by $-8 < x < -2$.

(d) The solutions of the inequality $(x-1)(x-2)(x-3) \geq 27x - 6$ are given by $-2 \leq x \leq 0$ or $x \geq 8$.

(e) The solutions of the inequality $(x-1)^2(x-4) \geq 0$ are given by $x = 1$ or $x \geq 4$.

(f) The solutions of the inequality $(x-1)(x-3)^2 \leq 0$ are given by $x \leq 1$ or $x = 3$.

(g) The solutions of the inequality $(x+3)x(x-1)(x-2) > 0$ are given by $x < -3$ or $0 < x < 1$ or $x > 2$.

(h) The solutions of the inequality $(x-1)(x-2)(x-4)(x-8) \leq 0$ are given by $1 \leq x \leq 2$ or $4 \leq x \leq 8$.

8. (a) The solutions of the inequality $x > -\dfrac{5}{x} + 6$ are given by $0 < x < 1$ or $x > 5$.

(b) The solutions of the inequality $x \leq -\dfrac{6}{x+1} + 4$ are given by $x < -1$ or $1 \leq x \leq 2$.

(c) The solutions of the inequality $2x - 1 \leq \dfrac{3}{x-1} - 4$ are given by $x \leq -2$ or $1 < x \leq 1.5$.

(d) The solutions of the inequality $\dfrac{2x}{x+1} \geq 2x - 1$ are given by $-1 < x \leq -0.5$ or $x \geq 1$.

(e) The solutions of the inequality $\dfrac{2x-3}{x+1} \leq 1$ are given by $-1 < x \leq 4$.

(f) The solutions of the inequality $\dfrac{3x+1}{x+2} \geq 1$ are given by $x < -2$ or $x \geq 3$.

(g) The solutions of the inequality $\dfrac{1}{x+1} \leq \dfrac{1}{3-x}$ are given by $x < -1$ or $1 \leq x < 3$.

(h) The solutions of the inequality $\dfrac{1}{x^2 - 6x + 8} \geq 0$ are given by $x < 2$ or $x > 4$.

(i) The solutions of the inequality $\dfrac{3}{x^2 - 6x + 8} \geq 1$ are given by $1 \leq x < 2$ or $4 < x \leq 5$.

(j) The solutions of the inequality $\dfrac{x^2 - 7x + 12}{x^2 - 3x + 2} \leq 0$ are given by $1 < x < 2$ or $3 \leq x \leq 4$.

(k) The solutions of the inequality $\dfrac{x^2 - 7x + 12}{x^2 - 3x + 2} \leq -1$ are given by $1 < x < 2$.

(l) The solutions of the inequality $\dfrac{x^2 - 1}{x^2 - 4} \geq 0$ are given by $x < -2$ or $-1 \leq x \leq 1$ or $x > 2$.

(m) The solutions of the inequality $\dfrac{x^2 - 1}{x^2 - 4} \geq 1$ are given by $x < -2$ or $x > 2$.

9. (a) The solutions of the inequality $|x + 3| < 2$ is given by $-5 < x < -1$.

(b) The solutions of the inequality $|2x - 9| \leq 15$ is given by $-3 \leq x \leq 12$.

(c) The solutions of the inequality $|8 - 3x| \leq 7$ is given by $\dfrac{1}{3} \leq x \leq 5$.

(d) The solutions of the inequality $|x - 2| > 4$ is given by $x < -2$ or $x > 6$.

(e) The solutions of the inequality $|2x + 5| \geq 13$ is given by $x \leq -9$ or $x \geq 4$.

(f) The solutions of the inequality $|6 - x| \geq 6$ is given by $x \leq 0$ or $x \geq 12$.

(g) The solutions of the inequality $|x^2 + 7x - 1| < 7$ is given by $-8 < x < -6$ or $-1 < x < 1$.

(h) The solutions of the inequality $|2x^2 - 8x - 1| \leq 9$ is given by $-1 < x < 5$.

(i) The solutions of the inequality $|-x^2 + 2x + 3| \geq 5$ is given by $x \leq -2$ or $x \geq 4$.

(j) The solutions of the inequality $|x^2 - x - 3| < 3$ is given by $-2 < x < 0$ or $1 < x < 3$.

(k) The solutions of the inequality $\left|\dfrac{3x-1}{4x+1}\right| > 0$ is given by $x < -\dfrac{1}{4}$ or $-\dfrac{1}{4} < x < \dfrac{1}{3}$ or $x > \dfrac{1}{3}$.

(l) The solutions of the inequality $\big|\ 2|x| - 9\ \big| \leq 5$ is given by $-7 \leq x \leq -2$ or $2 \leq x \leq 7$.

(m) The solutions of the inequality $x^2 < |x+2|$ is given by $-1 < x < 2$.

(n) The solutions of the inequality $|3x+1| \geq x^2 + 1$ is given by $-2 \leq x \leq -1$ or $0 \leq x \leq 3$.

(o) The solutions of the inequality $\dfrac{|x-3|}{2x} < 1$ is given by $x < 0$ or $x > 1$.

(p) The solutions of the inequality $\dfrac{|x-9|}{3x+1} > 1$ is given by $x > 2$.

(q) The solutions of the inequality $|4x+1| > |x-3|$ is given by $x < -\dfrac{4}{3}$ or $x > \dfrac{2}{5}$.

(r) The solutions of the inequality $(x+2)|x-2| < -5$ is given by $x < -3$.

(s) The solutions of the inequality $x^2 - |x| - x < 0$ is given by $0 < x < 2$.

10. (a) The solutions of the inequality are $\sqrt{4x+1} < x+1$ given by $-\dfrac{1}{4} \leq x < 0$ or $x > 2$.

(b) The solutions of the inequality $\sqrt{6x+3} > 3x+1$ are given by $-\dfrac{1}{2} \leq x < \dfrac{\sqrt{2}}{3}$.

11. (a) $x = \dfrac{c+1}{c}$.

(b) $(\star_0)$ has no solution.

12. (a) $x = c+1$.

(b) Every real number is a real solution of $(\star_0)$.

13. (a) The only solution of $(\star_{a,b})$ is given by $x = \dfrac{b-2}{a^2-4a+3}$.

(b) i. $(a,b) = (1,2)$ or $(a,b) = (3,2)$.

ii. When $(a,b) = (1,2)$ or $(a,b) = (3,2)$, every real number is a solution of $(\star_{a,b})$.

14. (a) The only solution of $(\star_c)$ is given by $x = \dfrac{c}{c-1}$.

(b) $(\star_c)$ has a real solution iff $c > 1$.

15. (a) $c = 4$.

(b) The only solution of the system $(\star_c)$ is given by $(x,y) = (-1,2)$.