MATH1050 Examples: Equations involving trigonometric functions.

1. For each equation with unknown in the reals below, determine its solution set by solving for its general solution.

   You are not required to give the 'checking step' explicitly, but be careful not to wrongly include false candidates amongst the solution, nor wrongly ignore a genuine solution.

   (a) $\sin(2x) + \cos(x) = 0$.
   (b) $\tan^2(x) + 3 = 2\sec^2(x)$.
   (c) $\cos(3x) = \cos(x)$.
   (d) $\sin(x) + \sin(2x) + \sin(3x) = 0$.
   (e) $\sin(2x) + \sin(4x) = \cos(x)$.
   (f) $\cos(4x) + \cos(2x) = \cos(x)$.
   (g) $2\cos(2x) + 5\sin(x) - 3 = 0$.
   (h) $\sin(5x) + \sin(3x) = \cos(x)$.
   (i) $12\cos(3x) - 5\sin(3x) = 13$.
   (j) $\sin(3x + \frac{\pi}{4})\cos(3x - \frac{\pi}{4}) = \frac{3}{4}$.
   (k) $\cos(4x) - 2\sin^2(x) = -2\sin^2(\frac{x}{2})$.
   (l) $\sin(\frac{2}{x}) = \frac{1}{2}$.
   (m) $\cot(\frac{x^3}{3}) = -\sqrt{3}$.
   (n) $\cos(4x^2) = -1$.
   (o) $\sin(\frac{1}{\sqrt{x}})\sin(x^2) = 0$.
   (p) $\sin(\frac{2}{x}) = \sin(\frac{1}{x})$.

2. Determine the solution set of the equation $\sin^2(3\theta) - \sin^2(2\theta) - \sin(\theta) = 0$.

   Remark. Express $\sin^2(\mu) - \sin^2(\nu)$ in terms of $\cos(\mu+\nu), \cos(\mu-\nu), \sin(\mu+\nu), \sin(\mu-\nu)$.

3. Determine the solution set of the equation $\cos^2(2\theta) - \sin^2(3\theta) + \cos(\theta)\sin(5\theta) = 0$.

   Remark. Express $\cos^2(\mu) - \sin^2(\nu)$ in terms of $\cos(\mu+\nu), \cos(\mu-\nu), \sin(\mu+\nu), \sin(\mu-\nu)$.

4. Determine the solution set of the equation $\sin(4x) - \sin(3x) + \sin(2x) - \sin(x) = 0$.

   Remark. Express $\sin(4\theta) - \sin(3\theta) + \sin(2\theta) - \sin(\theta)$ in the form $A\sin(\frac{\theta}{2})\cos(p\theta)\cos(q\theta)$. Here A, p, q are some real numbers whose values you have to determine.

5. Determine the solution set of the equation $(4\cos^2(x) - 3)\sin(2x) = \sin(x)$.

   Remark. Express $\cos(3\theta)$ in terms of $\cos(\theta)$.

6. (a) Let $\alpha, \beta, k \in \mathbb{R}$. Suppose $\tan(\alpha), \tan(\beta)$ are well-defined as real numbers. Further suppose that $\tan(\alpha) = k\tan(\beta)$.
      i. Prove that $\sin(\alpha+\beta) = (k+1)\cos(\alpha)\sin(\beta)$.
      ii. Hence deduce that $(k+1)\sin(\alpha-\beta) = (k-1)\sin(\alpha+\beta)$.

   (b) Let $\theta, k \in \mathbb{R}$. Suppose $\tan(\theta + \frac{\pi}{18}), \tan(\theta - \frac{\pi}{9})$ are well-defined as real numbers and $\tan(\theta + \frac{\pi}{18}) = k\tan(\theta - \frac{\pi}{9})$.
      i. Prove that $\sin(2\theta - \frac{\pi}{18}) = \frac{k+1}{2(k-1)}$.
      ii. Determine all possible values of k.

   (c) Determine the solution set of the equation $\tan(x + \frac{\pi}{18}) = -2\tan(x - \frac{\pi}{9})$.