

MATH1050 Guided Study Exercise 3

Advice.

- This guided study exercise is associated with the handouts *Image sets and pre-image sets*, *Image sets and pre-image sets under 'nice' real-valued functions of one real variable*. It is intended that you study those handouts first before attempting the questions below.

All questions below are about reading off image sets and pre-image sets for a function, or some other related information, according to the respective definitions for the notions of image sets, pre-image sets. In each question, drawing an appropriate diagram ('blobs-and-arrows diagram' or 'coordinate-plan diagramm') which carries the information of the function concerned will help.

1. Let A be the subset of \mathbb{N} defined by $A = \{0, 1, 2, 3, 4, 5\}$, and $f : A \rightarrow A$ be the function defined by $f(0) = 1$, $f(1) = 1$, $f(2) = 2$, $f(3) = 2$, $f(4) = 2$, $f(5) = 5$.

Consider each of the sets below. Where it is not the empty set, list every element of the set concerned, each element exactly once. Where it is the empty set, write 'it is the empty set'. You are not required to justify your answer.

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| (a) $f(\{1, 2, 3\})$ | (d) $f(\{0, 2, 4\})$ | (g) $f^{-1}(f(\{0, 2, 4\}))$ |
| (b) $f^{-1}(\{1\})$ | (e) $f^{-1}(\{0, 2, 4\})$ | (h) $(f \circ f)(\{0, 2, 4\})$ |
| (c) $f^{-1}(\{3, 4\})$ | (f) $f(f^{-1}(\{0, 2, 4\}))$ | (i) $(f \circ f)^{-1}(\{0, 2, 4\})$ |

2. Let $f : \mathbb{R} \setminus \{0, 2\} \rightarrow \mathbb{R}$ be the function defined by $f(x) = \frac{2}{x(x-2)} + 1$ for any $x \in \mathbb{R} \setminus \{0, 2\}$.

Write down the respective values of the numbers $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \kappa, \lambda, \mu, \nu, \xi, \rho, \sigma, \tau, \varphi, \psi, \omega$, so that the set equalities below hold. You are not required to justify your answer.

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| (a) $f((0, 2)) = (-\infty, \alpha]$. | (e) $f^{-1}([3, 9]) = [\theta, \kappa] \cup [\lambda, \mu]$. |
| (b) $f([4, +\infty)) = (\beta, \gamma]$. | (f) $f^{-1}([-3, 0]) = [\nu, \xi]$. |
| (c) $f((1, 3) \setminus \{2\}) = (-\infty, \delta) \cup (\varepsilon, +\infty)$. | (g) $f^{-1}([-3, 1]) = [\rho, \sigma]$. |
| (d) $f^{-1}(\{3\}) = \{\zeta, \eta\}$. | (h) $f^{-1}([-3, 3]) = (-\infty, \tau] \cup [\varphi, \psi] \cup [\omega, +\infty)$. |

3. Define the function $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x^{-2} & \text{if } x < -1 \\ -1 & \text{if } x = -1 \\ -x & \text{if } -1 < x < 0 \\ 2 & \text{if } x = 0 \\ 2x^2 + 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } x = 1 \\ 1 + x^{-1} & \text{if } x > 1 \end{cases}.$$

Write down the respective values of the numbers $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \kappa$, so that the set equalities below hold. You are not required to justify your answer.

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| (a) $f(\mathbb{R}) = ([\alpha, \beta] \setminus \{\gamma\}) \cup \{\delta\}$. | (b) $f^{-1}([0.25, 3]) = ([\varepsilon, \zeta] \setminus \{\eta\}) \cup ([\theta, +\infty) \setminus \{\kappa\})$. |
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4. You are not required to justify your answers in this question.

Let $a, b \in \mathbb{R}$, and $f : [0, 5] \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} -\frac{12}{(x+1)(x-3)} & \text{if } 0 \leq x < 3 \\ a & \text{if } x = 3 \\ -(x-3)(x-5) & \text{if } 3 < x \leq 5 \text{ and } x \neq 4 \\ b & \text{if } x = 4 \end{cases}.$$

Suppose $f(3) < f(4)$. Further suppose that $f^{-1}(\{2\}) \neq \emptyset$ and $f^{-1}(\{3\})$ has exactly two elements.

- What are the respective values of a, b ?
- Name all two elements of $f^{-1}(\{3\})$.
- What are the numbers $\alpha, \beta, \gamma, \delta$ for which the set equality $f([2, 4]) = (\alpha, \beta) \cup \{\gamma\} \cup [\delta, +\infty)$ holds?