MATH1050 Guided Study Exercise 2 (Answers)

1. **Answer.**

   (a) $(\exists a)(\forall b)(\exists c)[P(c) \wedge [(\sim Q(a)) \vee (\sim R(b))]]$.

   (b) $(\forall a)(\exists b)(\forall c)[(\sim P(a,c)) \vee [Q(a,b,c) \wedge (\sim R(a,b,c))]]$.

   (c) $(\exists a)(\exists b)(\forall c)(\forall d)[[(\sim P(a,c)) \wedge (\sim Q(b,d))] \vee (\sim R(c,d))]$.

   (d) $(\exists a)(\exists b)[P(a,b) \wedge [(\forall c)(\exists d)[(\sim Q(a,b,c,d)) \vee (\sim R(a,b,c,d))]]]$.

   (e) $(\exists a)[[(\exists b)(\forall c)P(b,c) \longrightarrow Q(a,b,c)] \wedge [(\exists d)(\forall e)[R(a,d,e) \wedge [(\sim S(a,d,e)) \vee (\sim T(a,d,e))]]]]$.

2. **Answer.**

   (a) There exists some $x \in \mathbb{R}$ such that for any $s,t \in \mathbb{Q}$, there exists some $n \in \mathbb{Z}$ such that $s < n < t$ and $|x-n| \leq |t-s|$.

   (b) There exists some $p \in \mathbb{R}$, such that for any $q \in \mathbb{R}$, $n \in \mathbb{N}$, there exist some $s,t \in \mathbb{R}$ such that $|s-t| < |q|$ and $|s^n - t^n| \geq |p|$.

   (c) There exists some $s,t \in \mathbb{Q}$ such that for any $p,q \in \mathbb{R}$, there exists some $n \in \mathbb{Z}$ such that $|s-t| \leq |q|$ and ($t^n > |p|$ or $s^n > |p|$).

   (d) For any $n \in \mathbb{N}$, there exists some $\varepsilon \in (0,+\infty)$ such that for any $\delta \in (0,+\infty)$, there exist some $u,v \in \mathbb{C}$ such that $|u-v| < \delta$ and $|u^n - v^n| \geq \varepsilon$.

   (e) There exist some $p,q \in \mathbb{Z}$ such that for any $s,t \in \mathbb{Z}$, there exist some $m,n \in \mathbb{N}$ such that $|p+q| \geq s^m$ and $|p^n - q| \geq t$ and $|p - q^n| \geq t$.

   (f) There exists some $z \in \mathbb{C}$ such that for any $r \in \mathbb{R}$, there exists some $w \in \mathbb{C}$ such that ($|z-w| \leq r$ and ($z \in \mathbb{R}$ or $|w| \leq r$)).

   (g) There exist some $z,w \in \mathbb{C}$ such that $|z-w| \geq |z+w|$ and (for any $s \in \mathbb{R}$, there exists some $t \in \mathbb{R}$ such that ($|z-s-t| \leq w$ and $|z| \geq 1$)).

   (h) There exist some $\zeta, \alpha, \beta \in \mathbb{C}$ such that (there exist some $s,t \in \mathbb{R}$ such that $\zeta = s\alpha + t\beta$) and (for any $p,q \in \mathbb{R}$, $\zeta \neq p\bar{\alpha} + q\bar{\beta}$.)

3. **Answer.**

   (a) There exist some $u,v,w \in \mathbb{R}$ such that $u+v > w$ and (for any $s,t \in \mathbb{R}$, $su+tv < w$).

   (b) For any $u,v,w \in \mathbb{R}$, there exists some $s,t \in \mathbb{R}$ such that $su+tv < w$ and ($s^2u + t^2v \geq w^2$ or $s^3u + t^3v \geq w^3$).

   (c) There exist some $u,v,w \in \mathbb{R}$ such that $u+v < w$ and (there exist some $s,t \in \mathbb{R}$ such that for any $r \in \mathbb{R}$, $su+tv \geq rw$).

   (d) For any $u,v,w \in \mathbb{R}$, (there exist some $r,s,t \in \mathbb{R}$ such that $ru^2 + sv^2 + tw^2 > uvw$) or (there exist some $r,s,t \in \mathbb{R}$ such that $r^2u + s^2v + t^2w < uvw$.

   (e) There exist some $u,v,w \in \mathbb{R}$ such that (for any $r,s,t \in \mathbb{R}$, $ru^2 + sv^2 + tw^2 > 0$) and (there exist some $p,q \in \mathbb{R}$ such that $r^2u + s^2v + t^2w \geq pq$).

   (f) There exist some $u,v,w \in \mathbb{R}$ such that (for any $s,t \in \mathbb{R}$, if $su+tv \leq w$ then $su^2 + tv^2 \geq w^2$) and (there exist some $p,q,r \in \mathbb{R}$ such that $(u+p)(v+q) \leq rw$ and $(u+p)^2 + (v+q)^2 < r^2w^2$).