MATH1050 Guided Study Exercise 2

Advice.

- This guided study exercise is associated with the handouts *Universal quantifier and existential quantifier*, *Statements with several quantifiers*. It is intended that you study those handout first before attempting the questions below.

  All the questions are concerned negating statements with several quantifiers. Although this looks like being about connected with a 'small portion' of those two handouts, it is the rest of the handouts which provides the groundwork the 'recipe' for correctly negating statements with several quantifiers described in that 'small portion' of the handouts.

- When you want to negate a statement which is made up of several inter-connected sentences, it may help by first re-formulating the statement concerned into a 'one-sentence statement' in which the quantifiers implicit in the original formulation are displayed explicitly, and in which the conditionals involved are displayed explicitly.

- You may also have to recall the logical equivalences concerned with the logical connective 'negation'. (Refer to the handout *Basics of logic in mathematics*.)

1. Consider each of the statements below. Write down its negation in such a way that the negation symbol only appears immediately to the left of the symbols standing for the 'atomic' predicates such as $P(x)$, $Q(y)$, $R(z)$, $S(x,y)$, $T(x,y,z)$.

   (a) $(\forall a)(\exists b)(\forall c)[P(c) \longrightarrow [Q(a) \wedge R(b)]]$.

   (b) $(\exists a)(\forall b)(\exists c)[P(a,c) \wedge [Q(a,b,c) \longrightarrow R(a,b,c)]]$.

   (c) $(\forall a)(\forall b)(\exists c)(\exists d)[[P(a,c) \vee Q(b,d)] \wedge R(c,d)]$.

   (d) $(\forall a)(\forall b)[P(a,b) \longrightarrow [(\exists c)(\forall d)[Q(a,b,c,d) \wedge R(a,b,c,d)]]]$.

   (e) $(\forall a)[[(\exists b)(\forall c)P(b,c) \longrightarrow Q(a,b,c)] \longrightarrow [(\forall d)(\exists e)[R(a,d,e) \longrightarrow (S(a,d,e) \wedge T(a,d,e))]]]$.

2. Consider each of the statements below. (Do not worry about the mathematical content.) Write down its negation in such a way that the word '*not*' does not explicitly appear.

   (a) *For any $x \in \mathbb{R}$, there exist some $s,t \in \mathbb{Q}$ such that for any $n \in \mathbb{Z}$, if $s < n < t$ then $|x-n| > |t-s|$.*

   (b) *For any $p \in \mathbb{R}$, there exists some $q \in \mathbb{R}$, $n \in \mathbb{N}$ such that for any $s,t \in \mathbb{R}$, if $|s-t| < |q|$ then $|s^n - t^n| < |p|$.*

   (c) *For any $s,t \in \mathbb{Q}$, there exist some $p,q \in \mathbb{R}$ such that for any $n \in \mathbb{Z}$, (if $|s-t| \leq |q|$ then ($t^n \leq |p|$ and $s^n \leq |p|$)).*

   (d) *There exists some $n \in \mathbb{N}$ such that (for any $\varepsilon \in (0,+\infty)$, there exists some $\delta \in (0,+\infty)$ such that (for any $u,v \in \mathbb{C}$, if $|u-v| < \delta$ then $|u^n - v^n| < \varepsilon$)).*

   (e) *For any $p,q \in \mathbb{Z}$, there exist some $s,t \in \mathbb{Z}$ such that for any $m,n \in \mathbb{N}$, if $|p+q| \geq s^m$ then ($|p^n - q| < t$ or $|p-q^n| < t$).*

   (f) *For any $z \in \mathbb{C}$, there exists some $r \in \mathbb{R}$ such that for any $w \in \mathbb{C}$, (if $|z-w| \leq r$ then ($z \notin \mathbb{R}$ and $|w| > r$)).*

   (g) *For any $z,w \in \mathbb{C}$, if $|z-w| \geq |z+w|$ then (there exists some $s \in \mathbb{R}$ such that (for any $t \in \mathbb{R}$, ($|z-s-t| > |w|$ or $|z| < 1$))).*

   (h) *For any $\zeta, \alpha, \beta \in \mathbb{C}$, if (there exist some $s,t \in \mathbb{R}$ such that $\zeta = s\alpha + t\beta$) then (there exist some $p,q \in \mathbb{R}$ such that $\zeta = p\bar{\alpha} + q\bar{\beta}$).*

3. Consider each of the statements below. (Do not worry about the mathematical content.) Write down its negation in such a way that the word '*not*' does not explicitly appear.

   (a) *For any $u,v,w \in \mathbb{R}$, if $u+v > w$ then there exist some $s,t \in \mathbb{R}$ such that $su+tv \leq w$.*

   (b) *There exist some $u,v,w \in \mathbb{R}$ such that for any $s,t \in \mathbb{R}$, if $su+tv < w$ then ($s^2u+t^2v < w^2$ and $s^3u+t^3v < w^3$).*

   (c) *Let $u,v,w \in \mathbb{R}$. Suppose $u+v < w$. Then for any $s,t \in \mathbb{R}$, there exists some $r \in \mathbb{R}$ such that $su+tv < rw$.*

   (d) *There exist some $u,v,w \in \mathbb{R}$ such that (for any $r,s,t \in \mathbb{R}$, $ru^2+sv^2+tw^2 \leq uvw$) and (for any $r,s,t \in \mathbb{R}$, $r^2u+s^2v+t^2w \geq uvw$).*

(e) *Let* $u, v, w \in \mathbb{R}$. *Suppose that for any* $r, s, t \in \mathbb{R}$, $ru^2 + sv^2 + tw^2 > 0$. *Then for any* $p, q \in \mathbb{R}$, $r^2u + s^2v + t^2w < pq$.

(f) *Let* $u, v, w \in \mathbb{R}$. *Suppose that for any* $s, t \in \mathbb{R}$, *if* $su + tv \leq w$ *then* $su^2 + tv^2 \geq w^2$. *Then for any* $p, q, r \in \mathbb{R}$, *if* $(u + p)(v + q) \leq rw$ *then* $(u + p)^2 + (v + q)^2 \geq r^2w^2$.