MATH1050 Guided Study Exercise 1 (Answers)

1. **Answer.**

(a) The statement $(P \to R) \to [(P \to Q) \wedge (Q \to R)]$ is neither a tautology nor a contradiction; it is a contingent statement.

P	Q	R	$P\to Q$	$Q\to R$	$P\to R$	$(P\to Q)\wedge(Q\to R)$	$(P\to R) \to [(P\to Q)\wedge(Q\to R)]$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	F
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Remark. Do you see what is wrong in the logic behind the word 'therefore' in the argument below?

- If John visits his girlfriend then John puts aside his books. Therefore, if John visits his girlfriend then John plays football; furthermore, if John plays football then John puts aside his books.

(b) The statement $(P \to Q) \to [(P \to R) \vee (Q \to R)]$ is neither a tautology nor a contradiction; it is a contingent statement.

P	Q	R	$P\to Q$	$P\to R$	$Q\to R$	$(P\to R)\vee(Q\to R)$	$(P\to Q) \to [(P\to R)\vee(Q\to R)]$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

2. **Answer.**

(a)

P	Q	$P\to Q$	$(P\to Q)\to Q$	$Q\to P$	$Q\to (Q\to P)$	$\sim P$	$(P\to Q)\to(\sim P)$	$Q\to(\sim P)$	$P\to[Q\to(\sim P)]$
T	T	T	T	T	T	F	F	F	F
T	F	F	T	T	T	F	T	T	T
F	T	T	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T	T	T

(b) U is a contingent statement.

(c) V is a contingent statement.

(d) W is a contingent statement.

(e) X is a contingent statement.

(f) $U \wedge V$ is logically equivalent to P.

(g) $W \longleftrightarrow X$ is a tautology.

(h) $U \longleftrightarrow W$ is logically equivalent to $\sim(P \longleftrightarrow Q)$.

3. **Answer.**

Let P, Q be statements. The statements $P \longleftrightarrow (\sim Q)$, $(\sim P) \longleftrightarrow Q$, $(P \vee Q) \wedge [\sim(P \wedge Q)]$, $[P \wedge (\sim Q)] \vee [(\sim P) \wedge Q]$ are logically equivalent to each other.

Here are two truth tables which display the truth values of P, Q and the statements $P \leftrightarrow (\sim Q)$, $(\sim P) \leftrightarrow Q$, $(P \vee Q) \wedge [\sim(P \wedge Q)]$, $[P \wedge (\sim Q)] \vee [(\sim P) \wedge Q]$:

P	Q	$\sim P$	$\sim Q$	$P \leftrightarrow (\sim Q)$	$(\sim P) \leftrightarrow Q$	$P \vee Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$(P \vee Q) \wedge [\sim(P \wedge Q)]$
T	T	F	F	F	F	T	T	F	F
T	F	F	T	T	T	T	F	T	T
F	T	T	F	T	T	T	F	T	T
F	F	T	T	F	F	F	F	T	F

P	Q	$\sim P$	$\sim Q$	$P \wedge (\sim Q)$	$(\sim P) \wedge Q$	$[P \wedge (\sim Q)] \vee [(\sim P) \wedge Q]$	$P \leftrightarrow (\sim Q)$
T	T	F	F	F	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	F	F	F

4. **Answer.**

   Let P, Q, R, S be statements, and denote the statements $(P \longrightarrow Q) \longrightarrow (R \longrightarrow S)$, $[P \longrightarrow (Q \longrightarrow R)] \longrightarrow S$ by U, V.

   Suppose the truth value of U is F.

   Then the truth value of $P \longrightarrow Q$ is T and the truth value of $R \longrightarrow S$ is F.

   Since the truth value of $R \longrightarrow S$ is F, the truth value of R is T and the truth value of S is F.

   Since the truth value of R is T, the truth value of $Q \longrightarrow R$ is T.

   Since the truth value of $Q \longrightarrow R$ is T, the truth value of $P \longrightarrow (Q \longrightarrow R)$ is T.

   Recall that the truth value of S is F. Then the truth value of V is F.

   Since the truth values of U, V are both F, the truth value of $V \longrightarrow U$ is T.

5. **Answer.**

   We verify that the truth value of $[(P \vee Q) \longrightarrow (Q \wedge R)] \longrightarrow (P \longrightarrow R)$ is T irrespective of the respective truth values of P, Q, R:

   - Suppose it happened that the truth value $[(P \vee Q) \longrightarrow (Q \wedge R)] \longrightarrow (P \longrightarrow R)$ were F for some specific truth values of P, Q, R respectively.

     Then for the same truth values of P, Q, R, it would happen that the truth value of $(P \vee Q) \longrightarrow (Q \wedge R)$ was T and the truth value of $P \longrightarrow R$ was F.
     Since the truth value of $P \longrightarrow R$ was F, the truth value of P was T and the truth value of R was F.
     Since the truth value of R was F, the truth value of $Q \wedge R$ was F.
     Since the truth value of P was T, the truth value of $P \vee Q$ was T.
     Then the truth value of $(P \vee Q) \longrightarrow (Q \wedge R)$ would be F. This is impossible.