MATH1050 Guided Study Exercise 1

Advice.

- This guided study exercise is associated with the handout *Basics of logic in mathematics*. It is intended that you study that handout first before attempting the questions below.

  All the questions are concerned with the logical connectives 'negation', 'conjunction', 'disjunction', 'conditional', 'biconditional', and the respective notions of tautology, contradiction, contingent statement.

1. Let P, Q, R be statements. Consider each of the statements below. Determine whether it is a tautology or a contradiction or a contingent statement . Justify your answer by drawing an appropriate truth table.

   (a) $(P \longrightarrow R) \longrightarrow [(P \longrightarrow Q) \wedge (Q \longrightarrow R)]$

   (b) $(P \longrightarrow Q) \longrightarrow [(P \longrightarrow R) \vee (Q \longrightarrow R)]$

2. Let P, Q be statements. Denote the compound statements

   $$(P \longrightarrow Q) \longrightarrow Q, \qquad Q \longrightarrow (Q \longrightarrow P), \qquad (P \longrightarrow Q) \longrightarrow (\sim P), \qquad P \longrightarrow [Q \longrightarrow (\sim P)]$$

   by U, V, W, X respectively.

   (a) Draw a truth table which displays the truth values of P, Q, U, V, W, X in all possible scenarios.

   (b) Is U a tautology, or a contradiction, or a contingent statement?

   (c) Is V a tautology, or a contradiction, or a contingent statement?

   (d) Is W a tautology, or a contradiction, or a contingent statement?

   (e) Is X a tautology, or a contradiction, or a contingent statement?

   (f) Is $U \wedge V$ logically equivalent to P, or to Q, or to both, or to neither?

   (g) Is $W \longleftrightarrow X$ a tautology, or a contradiction, or a contingent statement?

   (h) Is $U \longleftrightarrow W$ logically equivalent to $P \longleftrightarrow Q$, or to $\sim(P \longleftrightarrow Q)$, or to neither?

3. Let P, Q be statements. By drawing one or more appropriate truth tables, verify that the statements

   $$P \longleftrightarrow (\sim Q), \quad (\sim P) \longleftrightarrow Q, \quad (P \vee Q) \wedge [\sim(P \wedge Q)], \quad [P \wedge (\sim Q)] \vee [(\sim P) \wedge Q]$$

   are logically equivalent to each other.

   Remark. $(P \vee Q) \wedge [\sim(P \wedge Q)]$ is called the **exclusive disjunction** of P, Q. We may denote it by $P \veebar Q$. It is true exactly when one and only one of P, Q is true. In words we write 'P xor Q', or as 'either P or Q'. It is the kind of 'or' that you find in 'coffee or tea' in a restaurant menu.

4. Let P, Q, R, S be statements, and denote the statements $(P \longrightarrow Q) \longrightarrow (R \longrightarrow S)$, $[P \longrightarrow (Q \longrightarrow R)] \longrightarrow S$ by U, V.

   Suppose the truth value of U is F.

   What is the truth value of $V \longrightarrow U$? Justify your answer. (It is of course possible to make use of a truth table. However, try to give an argument without using a truth table.)

5. Let P, Q, R be statements.

   Determine whether $[(P \vee Q) \longrightarrow (Q \wedge R)] \longrightarrow (P \longrightarrow R)$ is a rule of inference. Justify your answer. (It is of course possible to make use of a truth table. However, try to give an argument without using a truth table.)