MATH1050 Assignment 11 (Answers and selected solutions)

1. **Answer.**
   $(s,t) = (0,0)$ and $(u,v) = (1,0)$.

2. **Answer.**
   $\alpha = 1$, $\beta = 2$, $\gamma = -1$, $\delta = 1$.

3. **Answer.**

   (I) for any $x \in \mathbb{R}$, $(x,x) \in E$

   (II) Pick any $x \in \mathbb{R}$

   (III) $x - x = 0$

   (IV) $(x,x) \in E$

   (V) for any $x, y \in \mathbb{R}$, if $(x,y) \in E$ then $(y,x) \in E$

   (VI) Suppose $(x,y) \in E$

   (VII) there exists some $a \in \mathbb{Q}$

   (VIII) $y - x = -(x-y) = -a$

   (IX) $a \in \mathbb{Q}$

   (X) $-a \in \mathbb{Q}$

   (XI) $(y,x) \in E$

   (XII) for any $x, y, z \in \mathbb{R}$, if $(x,y) \in E$ and (y,z) then $(x,z) \in E$

   (XIII) $x, y, z \in \mathbb{R}$

   (XIV) $(x,y) \in E$ and $(y,z) \in E$

   (XV) $x - y = a$

   (XVI) Since $(y,z) \in E$, there exists some $b \in \mathbb{Q}$ such that $y - z = b$

   (XVII) Since $a \in \mathbb{Q}$ and $b \in \mathbb{Q}$, we have $a + b \in \mathbb{Q}$

   (XVIII) $(x,z) \in E$

4. **Answer.**

   (a) (I) there exists some $x_0 \in \mathbb{R}$

   (II) $(x_0, x_0) \notin G$

   (III) Take $x_0 = 0$

   (IV) $x_0 - x_0$

   (V) $x_0 \cdot x_0$

   (VI) $(x_0, x_0) \notin G$

   (b) (I) there exists some $x_0, y_0 \in \mathbb{R}$

   (II) $(x_0, y_0) \in G$ and $(y_0, x_0) \notin G$

   (III) $x_0 = 1$, $y_0 = 0$

   (IV) $1 > 0$

   (V) $(x_0, y_0) \in G$

   (VI) $y_0 - x_0 > x_0 y_0$

   (VII) $(y_0, x_0) \notin G$

   (VIII) $(x_0, y_0) \in G$ and $(y_0, x_0) \notin G$

   (c) (I) there exists some $x_0, y_0, z_0 \in \mathbb{R}$

   (II) $(x_0, y_0) \in G$ and $(y_0, z_0) \in G$ and $(x_0, z_0) \notin G$

   (III) $z_0 = -2$

   (IV) $x_0 - y_0 = -5 > -6 = x_0 y_0$

(V) by the definition of G

(VI) $(y_0, z_0) \in G$

(VII) $x_0 - z_0 = 0 \leq 4 = x_0 z_0$

(VIII) $(x_0, z_0) \notin G$

(IX) $(x_0, y_0) \in G$ and $(y_0, z_0) \in G$ and $(x_0, z_0) \notin G$

5. **Answer.**

(a) (I) For any $\zeta \in \mathbb{C}^*$

(II) Pick any

(III) $\in \mathbb{C}^*$

(IV) $\frac{\zeta}{\zeta}$

(V) $\mathbb{R}^*$

(VI) $\in E$

(b) (I) if $(\zeta, \eta) \in E$ and $(\eta, \xi) \in E$ then $(\zeta, \xi) \in E$

(II) Suppose $(\zeta, \eta) \in E$ and $(\eta, \xi) \in E$.

(III) $\frac{\eta}{\zeta} \in \mathbb{R}^*$

(IV) $(\eta, \xi) \in E$

(V) $\frac{\xi}{\zeta} \in \mathbb{R}$

(VI) $\frac{\eta}{\zeta} \neq 0$ and $\frac{\xi}{\eta} \neq 0$

(VII) $\frac{\xi}{\zeta} \in \mathbb{R}^*$

(VIII) (ζ, ξ)

(c) (I) symmetric

(II) For any $\zeta, \eta \in \mathbb{C}^*$, if $(\zeta, \eta) \in E$ then $(\eta, \zeta) \in E$

(III) Pick any $\zeta, \eta \in \mathbb{C}^*$

(IV) $(\zeta, \eta) \in E$

(V) $\frac{\eta}{\zeta} \neq 0$

(VI) $\frac{\zeta}{\eta} \in \mathbb{R}$

(VII) $(\eta, \zeta) \in E$

(VIII) reflexive, symmetric and transitive

6. **Answer.**

Ω, Ξ, Π, Υ are partitions of A.

Σ, T, Γ, Δ are not partitions of A.

7. **Answer.**

The elements of E_Ω are: $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$, $(5, 5)$, $(1, 3)$, $(3, 1)$, $(1, 5)$, $(5, 1)$, $(3, 5)$, $(5, 3)$, $(2, 4)$, $(4, 2)$.

8. **Answer.**

(a) $(1, 1) \in E_f$

(b) $(1, -1) \in E_f$

(c) $(1, i) \notin E_f$

(d) $(2, \frac{1}{2}) \in E_f$

(e) $(2, \frac{1}{2}i) \notin E_f$

(f) $(2i, \frac{1}{2}i) \in E_f$

(g) $(2i, -\frac{1}{2}i) \in E_f$

(h) $(2,3) \notin E_f$

(i) $(1+i, 1-i) \in E_f$

(j) $(1-i, 1+i) \in E_f$

(k) $(4+3i, 4-3i) \in E_f$

(l) $(4+3i, 3+4i) \notin E_f$

9. (a) **Answer.**

(I) H is a subset of $J \times K$

(II) there exists some $y \in K$ such that $(x,y) \in H$

(III) $x \in J$

(IV) $y = 1 - x$

(V) $x \in J$

(VI) $y = 1 - x \leq 1 - 0 = 1$

(VII) $y = 1 - x > 1 - 1 = 0$

(VIII) $y \in K$

(IX) $(x,y) \in H$

(X) if $(x,y) \in H$ and $(x,z) \in H$ then $y = z$

(XI) Pick any $x \in J$, $y, z \in K$

(XII) $(x,y) \in H$ and $(x,z) \in H$

(XIII) Since $(x,y) \in H$

(XIV) $x + z = 1$

(XV) $z = 1 - x$

(XVI) $y = 1 - x = z$

(XVII) for any $y \in K$, there exists some $x \in J$

(XVIII) $(x,y) \in H$

(XIX) Pick any $y \in K$

(XX) $0 < y \leq 1$

(XXI) $y > 0$

(XXII) $x = 1 - y \geq 1 - 1 = 0$

(XXIII) $(x,y) \in H$

(XXIV) for any $x, w \in J$, for any $y \in K$

(XXV) then $x = w$

(XXVI) Suppose $(x,y) \in H$ and $(w,y) \in H$

(XXVII) Since $(x,y) \in H$

(XXVIII) $w + y = 1$

(XXIX) $x = 1 - y = w$

(XXX) h is a function

(XXXI) h is surjective

(XXXII) h is injective

(XXXIII) J is of cardinality equal to K

(b) **Solution.**

Let $L = (0,1)$, $M = \{0\}$, $N = \{1\}$.

Note that $J = L \cup M$, $K = L \cup N$, and $L \cap M = \emptyset$, $L \cap N = \emptyset$.

Let $D = \{(x,x) \mid x \in L\}$. The identity function id_L is a bijective function from L to L with graph D.

Note that $M \times N = \{(0,1)\}$. The relation $(M, N, M \times N)$ is a bijective function from M to N with graph $M \times N$.

Define $F = D \cup (M \times N)$, and define $f = (J, K, F)$.

By the Glueing Lemma, f is a bijective function from J to K with graph F.

It follows that $J \equiv K$.

10. **Answer.** *This is an outline of the full solution.*

Let $J = [0, 1)$, $L = (0, 1)$, $M = [0, +\infty)$, $N = (0, +\infty)$.

(a) A bijective function from J to M is $\varphi : J \longrightarrow M$, given by $\varphi(x) = -1 - \dfrac{1}{x-1}$ for any $x \in J$. It follows that $J{\sim}M$.

A bijective function from L to N is $\psi : L \longrightarrow N$, given by $\psi(x) = -1 - \dfrac{1}{x-1}$ for any $x \in L$. It follows that $L{\sim}M$.

(b) A bijective function from $\mathbb{R}$ to $(-1, 1)$ is $\alpha : \mathbb{R} \longrightarrow (-1, 1)$, given by $\alpha(x) = \dfrac{1 - e^{-x}}{1 + e^{-x}}$ for any $x \in \mathbb{R}$. It follows that $\mathbb{R}{\sim}(-1, 1)$.

A bijective function from $(-1, 1)$ to $(0, 1)$ is $\beta : (-1, 1) \longrightarrow (0, 1)$, given by $\beta(x) = \dfrac{x + 1}{2}$ for any $x \in (-1, 1)$. Now $\alpha^{-1} \circ \beta^{-1}$ a bijective function from L to $\mathbb{R}$. It follows that $L{\sim}\mathbb{R}$.