2 Complex form of Fourier series

Definition 7. A function $\phi(x)$ that is defined for $-\infty < x < \infty$ is called periodic if there is a number $p > 0$ such that $\phi(x+p) = \phi(x)$ for all x.

The smallest number p for which this is ture is called the period of $\phi(x)$.

If a function defined on the interval $-l < x < l$ its periodic extension is

$$\phi_{per}(x) = \phi(x - 2lm)$$

for $-l + 2lm < x < l + 2lm$ for all integers m.

The full Fouier series can be regarded either as an expansion of an arbitrary function on the interval $(-l, l)$ or as an expansion of a periodic function of period $2l$ defined on the whole line $-\infty < x < \infty$.

Definition 8. An even function is a function that satisfies $\phi(-x) = \phi(x)$.

An odd function is a function that satisfies $\phi(-x) = -\phi(x)$.

The Fourier sine (cosine) series can be regarded as an expansion of an arbitrary function that is odd (even) and has period $2l$ defined on the whole line $-\infty < x < \infty$.

Exercise 9. Prove that for $n \neq m$

$$\int_{-l}^{l} e^{in\pi x/l} e^{-im\pi x/l} dx = 0$$

and

$$\int_{-l}^{l} e^{i(n-n)\pi x/l} dx = 2l.$$

The complex form of the full Fourier series is

$$\phi(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/l}$$

where

$$c_n = \frac{1}{2l} \int_{-l}^{l} \phi(x) e^{-in\pi x/l} dx.$$

The advantage for complex form of Fourier series is that the complex form is somethimes more convenient in calculations than the real form with sines and cosines. But it really is just the same series written in a different form.