Integrals of the day: Part 4

1. Compute
$$\int_0^1 \frac{\ln(1+x)}{1+x^2}dx$$
Hint: Let I be the above integral. Substitute $x = \tan\theta$ to see that
$$I = \int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d\theta.$$
Substitute $t = \frac{\pi}{4} - \theta$ to see that
$$I = \ln 2 \int_0^{\frac{\pi}{4}} dt - I$$
so $I = \frac{\pi}{8}\ln 2$.

2. Compute
$$\int_0^1 (1-x^7)^{1/5} - (1-x^5)^{1/7}dx.$$
Hint: The answer is 0; one just needs to show
$$\int_0^1 (1-x^7)^{1/5}dx = \int_0^1 (1-y^5)^{1/7}dy.$$
But both are the area bounded by the curve $x^7 + y^5 = 1$ with the x and y-axes. So the two integrals are equal.

End