Integrals of the day: Part 3

1. Compute
$$\int \frac{1}{\sin(x-a)\sin(x-b)}dx$$
if $a - b$ is not a multiple of π.

Hint:
$$\int \frac{1}{\sin(x-a)\sin(x-b)}dx = \frac{1}{\sin(a-b)}\int \frac{\sin[(x-b)-(x-a)]}{\sin(x-a)\sin(x-b)}dx.$$

2. Compute
$$\int \sqrt{1+e^{2x}}dx.$$

Hint: Substitute $u = \sqrt{1+e^{2x}}$. Or else, substitute $v = e^x$. Then one arrives at the integral
$$\int \frac{\sec^3 x}{\tan x}dx,$$
which one could integrate by using
$$\int \frac{\sec^3 x}{\tan x}dx = \int \frac{\sec^3 x \tan x}{\tan^2 x}dx = \int \frac{\sec^2 x}{\sec^2 x - 1}d(\sec x).$$

The integrand in the last integral is a rational function in $\sec x$, which can be integrated using partial fractions.

End