MATH 4220 PDE-Quiz 2(10 points)

April 7, 2016

1. (**5 points**) Can the eigenvalue problem

$$\begin{cases} -X''(x) = \lambda X(x), & 0 < x < 1 \\ X'(0) = 0, & X(1) = 0 \end{cases}$$

   have nonpositive eigenvalues? Prove your statements. Write down all the eigenvalues and corresponding eigenfunctions.

2. (**5 points**) Find the Fourier cosine series of $f(x) = x$ on $(0, \pi)$. Then find the sum

$$\sum_{k=0}^{\infty} (\frac{1}{2k+1})^4 = 1 + (\frac{1}{3})^4 + (\frac{1}{5})^4 + (\frac{1}{7})^4 + \cdots$$

   by using Parseval's equality.