

THE CHINESE UNIVERSITY OF HONG KONG  
DEPARTMENT OF MATHEMATICS  
MATH2050 (First Term, 2012-2013)  
Mathematical Analysis I  
Homework I

Questions with \* will be marked. Deadline for Homework I: 28th Sept 5pm.

1. Let  $a, b \in \mathbb{R}$ . Show that

(a)\*  $a \cdot 0 = 0$ ;

(b)\*  $-a = (-1)a$ ;

(c)  $-(-a) = a$ ;

(d)  $(-a)(-b) = ab$ ;

(e)  $a^2 \geq 0$ ;

(f) If  $c < 0$  and  $a > b$  then  $ac < bc$ ;

(g) If  $a, b \geq 0$  then

$$a < b \Leftrightarrow a^2 < b^2 \Leftrightarrow \sqrt{a} < \sqrt{b},$$

where  $\sqrt{a}$  denotes the positive real number such that  $(\sqrt{a})^2 = a$ ; the existence of the square root is assumed and will be discussed later.

2. (a)\* Show that  $|x - a| < \varepsilon$  iff  $a - \varepsilon < x < a + \varepsilon$ .

(b) Find all  $x \in \mathbb{R}$  satisfying  $|x - 1| > |x + 1|$ .

3. Let  $A$  be a nonempty subset of  $\mathbb{R}$  and  $\ell \in \mathbb{R}$ . Give the definition for each of the following and the corresponding negation:

(a)\*  $\ell$  is a lower bound of  $A$ ;

(b)  $A$  is bounded below.

4. Let  $(x_n), (y_n)$  be sequences converge to  $x, y$  respectively. Show that

(a) There exist  $X, Y \in \mathbb{R}$  such that  $|x_n| \leq X$  and  $|y_n| \leq Y$  for all  $n \in \mathbb{N}$ ;

(b)\*  $\lim_{n \rightarrow \infty} (x_n + y_n) = x + y$ ;

(c)  $\lim_{n \rightarrow \infty} (x_n y_n) = xy$ .