THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 4050 Real Analysis

Tutorial 1 (February 1)

The following were discussed in the tutorial this week.

1. Recall the definition of outer measure m^* and its properties.
2. Recall the Carathéodory's criterion, measurable sets and Lebesgue measure m.
3. Recall the existence of a non-measurable set.
4. (Monotone Convergence Lemma for Measures) Let $\{E_n\}_{n\in\mathbb{N}}$ be a sequence of measure sets.
   (a) If $E_n \uparrow E$ (i.e. $E = \bigcup_{n\in\mathbb{N}} E_n$ and $E_n \subseteq E_{n+1}\ \forall\, n$), then $m(E_n) \uparrow m(E)$.
   (b) Suppose $m(E_{n_0}) < +\infty$ for some n_0. If $E_n \downarrow E$ (i.e. $E = \bigcap_{n\in\mathbb{N}} E_n$ and $E_n \supseteq E_{n+1}\ \forall\, n$), then $m(E_n) \downarrow m(E)$.
   (c) Can we drop the extra assumption in (b)?
5. Give an example of a sequence of set $\{E_n\}_{n=1}^{\infty}$ such that $m^*(E_1) < +\infty$, $E_n \downarrow E$ but $m^*(E) < \lim_n m^*(E_n)$.
6. Let $\{E_n\}_{n=1}^{\infty}$ be a sequence of set such that $E_n \uparrow E$. Show that $\lim_n m^*(E_n) = m^*(E)$.