Hw 7

$\S 5.1$ Q 12, 13, 15

Hints. Q12: Density of $\mathbb{Q}$ + seq. criterion

Q13. Let x_0 be a cts. pt. Take 2 sequences from $\mathbb{Q}$ and $\mathbb{R}\setminus\mathbb{Q}$ respectively & convergent to x_0.

Q15. Take a seq (x_n) in $(0,1)$ convergent to 0 such that $\lim_n f(x_n) = \ell \ (\in \mathbb{R})$. [such seq exists by B-W].

By assumption that $\lim_{x\to 0} f(x)$ not exist, $\exists$ a seq (z_n) convergent to 0 but $\lim_n f(z_n) \neq \ell$ — may assume without loss of generality that, for some $\varepsilon > 0$,

$$|f(z_n) - \ell| \geq \varepsilon \quad \forall n$$

(passing to a subseq, if nece). Apply B-W to get your (y_n).

$\S 5.2$ Q1 (Assuming the knowledge that $\sin x, \cos x$ etc cts

Hint: $x \mapsto \sqrt{x}$ is cts on $[0,\infty)$ & use results for combination of cts functions

$\S 5.3$ Q1, Q4, Q5.

Hint: Q1. Use the maximum/minimum value theo.

Q4. Let p be a poly of degree $2n+1$, and $q(x) = x^{2n+1}$.
By looking at $\lim_{x\to+\infty}$ & $\lim_{x\to-\infty}$ of $\frac{p(x)}{q(x)}$, $\exists$ $x_1 < x_2$ such that $p(x_1) < 0 < p(x_2)$.

Q5. Find $x_1 < x_2 < x_3$ such that the sign of p at x_2 is opposite to that for the other two pts.