

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Spring MATH2230
Tutorial 5

1. Find all values of $\log(\log i)$ in form of $x + yi$.
2. Find the image of the unit circle under the map $f = z + 1/z$.
3. Find the radius of the circle such that the image of that circle under the map $f = z + 1/z$ is the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = \frac{1}{4}$.
4. Show that $f = \arg z$ for $z \neq 0$ and $-\pi \leq \arg z < \pi$ is discontinuous on the negative real axis.
5. Define $f = z^{a+bi}$ on the principal branch where a and b are positive real number. Show that f is one to one if and only if $a^2 + b^2 < a$.
6. Calculate $\int_C \frac{z}{\bar{z}} dz$ along the positively oriented simple closed contour C which is the sum of four portions:
 - the upper half circle of $|z| = 2$,
 - the line segment $[-2, -1]$,
 - the upper half circle of $|z| = 1$,
 - the line segment $[1, 2]$.