THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050B Mathematical Analysis I

Tutorial 8 (November 4, 6)

The following problems were discussed in the tutorial this week:

Example 1. Use ε-δ definition to check that

(a) $\lim\limits_{x\to 0} \cos\left(\dfrac{1}{x}\right)$ does not exist;

(b) $\lim\limits_{x\to\infty} (-x^2 + \sin x) = -\infty$

Example 2. By ε-δ definition, show that $\lim\limits_{x\to 2} \dfrac{x^2+1}{x^2-3} = 5$.

Example 3. Let $D \subseteq \mathbb{R}$, $x_0 \in D^c$ and $f_i \colon D \to \mathbb{R}$ $(i = 1, 2)$ be such that $f_2(x) \neq 0$ for all $x \in D$. Suppose $\lim\limits_{x\to x_0} f_i(x) = \ell_i$ $(i = 1, 2)$ with $\ell_2 = 0$ and $\ell_1 \neq 0$. Show that, by definition,

(a) $\exists\, m \in (0, \infty)$ and $\delta_0 > 0$ such that $|f(x)| \geq m$ $\forall\, x \in V_{\delta_0}(x_0) \cap (D \setminus \{x_0\})$;

(b) $\lim\limits_{x\to x_0} \dfrac{|f_1(x)|}{|f_2(x)|} = +\infty$.

Example 4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} x + 2 & \text{if } x \in \mathbb{Q} \\ 3x - 1 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$. Exactly at what c does $\lim\limits_{x\to c} f(x)$ exist? And what is the limit then?

Example 5. Prove that if $f \colon \mathbb{R} \to \mathbb{R}$ is periodic and $\lim\limits_{x\to\infty} f(x) = 0$, then f is identically zero.