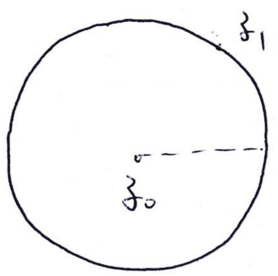
Lectures 16 and 17.

The lectures of this week is about Uniqness of Laurent Series expansion.

Fact 1: $S(z) = \sum_{n=0}^{+\infty} a_n (z-z_0)^n$.



if $S(z_1)$ is Convergent. then $\forall z$ satisfying $|z-z_0| < |z_1-z_0|$ $S(z)$ is also convergent

The disk $B_{|z_1-z_0|}(z_0)$ is called convergent disk.

Fact 2: $G(z) = \sum_{n=0}^{+\infty} a_n (z-z_0)^{-n}$.

if $G(z_1)$ is convergent, then

$\forall z$ satisfying $|z-z_0| > |z_1-z_0|$, $S(z)$ is convergent.

Therefore $S(z)$ has definition in the convergent disk.

Now we consider its continuity. Here we need <u>uniform convergence</u>

Def: f_n uniformly converges to f in Ω

$\Longleftrightarrow$ $\forall\, \varepsilon > 0$. $\exists\, N = N(\varepsilon)$. s.t $|f_n(\zeta) - f(\zeta)| < \varepsilon$

for all $\zeta \in \Omega$ and $n > N$.

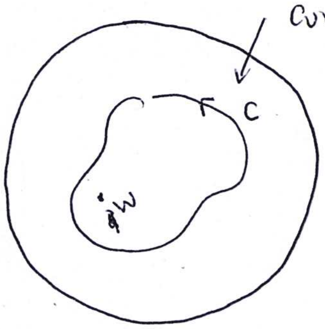
Here. N depends on ε only.

Uniform convergence implies continuity in the following sense.

Thm: f_n is continuous in Ω. and f_n converges to f uniformly. then f is also continuous.

In the next we study exchange of sum series and integration

convergent disk



$\forall\, g$ continuous on C.

it holds

$$\int_C g(\zeta)\, S(\zeta)\, d\zeta = \sum_{n=0}^{+\infty} a_n \int_C g(\zeta)(\zeta - \zeta_0)^n\, d\zeta .$$

Taking $g \equiv 1$

$$\Rightarrow \int_C S(z)\, dz = 0. \quad \Rightarrow \quad S \text{ is analytic in the}$$

convergent disk.

Taking $g = \dfrac{1}{(z-w)^2}$

$$\Rightarrow \quad S'(w) = \sum_{n=0}^{+\infty} n a_n (w - z_0)^{n-1}.$$

∴ differential and sum series can also be switched in convergent disk.

Finally we show that Laurent series representation of f is unique.