THE CHINESE UNIVERSITY OF HONG KONG
MATH2230 Tutorial 10

(Prepared by Tai Ho Man)

1. Let $f(z) = z^{a-1}$ where a is nonzero real number and C is postively oriented circle of radius R centred at origin. Find the value of $\int_C f dz$ in principal branch.

   Ans : $\frac{2iR^a \sin(a\pi)}{a}$.

2. Suppose f is analytic on $\Omega = \{z : 0 < |z| < 1\}$. and satisfies

$$|f(z)| \leq \log\left(\frac{1}{|z|}\right) \quad , \quad \text{for all } z \in \Omega.$$

   (a) Suppose f can be represented by the Laurent series,

$$f(z) = \sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} \frac{b_n}{z^n} \quad \text{for all } z \in \Omega.$$

   Show that $b_n = 0$ for $n = 1, 2, 3....$

   (b) Show that $a_n = 0$ for $n = 1, 2, 3....$

3. Let C be the counter-clockwisely oriented circle centred at $-i$ with radius 4. Compute the integral

$$\int_C \frac{\sin z}{(z+i)(z+3)^2} dz.$$

4. Let $f = u + iv$ be a continuous function on a closed bounded region R and analytic and not constant throughout the interior of R. Prove that u and v attain the maximum and minimum value on the boundary of R and never in the interior.

5. Find the series expansion of $\frac{1}{z^2}$ for $|z+1| < 1$.

   Ans : $1 + \sum_{1}^{\infty} (n+1)(z+1)^n$.