THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Fall MATH2230
Homework Set 7 (Due on Nov. 5)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

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1 Use an antiderivative to show that for every contour C extending from a point z_1 to a point z_2,

$$\int_C z^n dz = \frac{1}{n+1}(z_2^{n+1} - z_1^{n+1}) \ \text{(n=0,1,2...)}.$$

2 By finding an antiderivative, evaluate each of these integrals, where the path is any contour between the indicated limits of integration:

(a) $\int_0^{1+i} z^2 dz$; (b) $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$; (c) $\int_1^3 (z-2)^3 \, dz$.

3 Use the theorem in Sec. 48 to show that

$$\int_{C_0} (z - z_0)^{n-1} dz = 0 \quad (n = \pm 1, \pm 2...).$$

when C_0 is any closed contour which docs not pass through the point z_0.

5 Show that

$$\int_{-1}^{1} z^i dz = \frac{1+e^{-\pi}}{2}(1-i),$$

where the integrand denotes the principal branch

$$z^i = \exp(i\text{Log}z) \quad (\,|z| > 0, -\pi < \text{Arg}\, z < \pi)$$

of z^i and where the path of integration is any contour from $z = -1$ to $z = 1$ that except for its end points, lies above the real axis, for its endpoints, lies above the real axis.

Suggestion: Use an antiderivative of the branch

$$z^i = \exp(i \log z) \quad (\,|z| > 0, -\pi/2 < \arg z < 3\pi/2)$$

of the same power function.

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3 If C_0 denotes a positively oriented circle $|z - z_0| = R$, then

$$\int_{C_0} (z - z_0)^{n-1} dz = \begin{cases} 0 & \text{when } n = \pm 1, \pm 2... \\ 2\pi i & \text{when } \ n = 0. \end{cases}$$

according to Exercise 13. Sec. 46. Use that result and the corollary in Sec. 53 to show that if C is the boundary of the rectangle $0 \leq x \leq 3, 0 \leq y \leq 2$, described in the positive sense, then

$$\int_C (z-2-i)^{n-1} dz = \begin{cases} 0 & \text{when } n = \pm 1, \pm 2 ... \\ 2\pi i & \text{when } \ n = 0. \end{cases}$$