THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Fall MATH2230
Homework Set 1 (Due on)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P.13

3. Use established properties of moduli to show that when $|z_3| \neq |z_4|$,

$$\frac{Re(z_1 + z_2)}{|z_3 + z_4|} \leq \frac{|z_1| + |z_2|}{||z_3| - |z_4||}.$$

4. Verify that $\sqrt{2}|z| \geq |Rez| + |Imz|$.
Suggestion: Reduce this inequality to $(|x| - |y|)^2 \geq 0$.

5. In each case, sketch the set of points determined by the given condition:
(a) $|z - 1 + i| = 1$, (b)$|z + i| \leq 3$, (c) $|z - 4i| \geq 4$.

6. Using the fact that $|z_1 - z_2|$ is the distance between two points z_1 and z_2 give a geometric argument that $|z - 1| = |z + i|$ represents the line through the origin whose slope is -1.

P.16

1. Use properties of conjugates and moduli established in Sec. 6 to show that
(a) $\overline{\overline{z} + 3i} = z - 3i$, (b) $\overline{iz} = -i\overline{z}$, (c) $\overline{(2 + i)^2} = 3 - 4i$,
(d) $|(2\overline{z} + 5)(\sqrt{2} - i)| = \sqrt{3}|2z + 5|$.

2. Sketch the set of points determined by the condition
(a) $\text{Re}(\overline{z} - i) = 2$; (b) $|2\overline{z} + i| = 4$.

7. Show that

$$|Re(2 + \overline{z} + z^3)| \leq 4 \text{ when } |z| \leq 1$$

9. By factoring $z^4 - 4z^2 + 3$ into two quadratic factors and using inequality (2). Sec. 5. show that if z lies on the circle $|z| = 2$, then

$$\left|\frac{1}{z^4 - 4z^2 + 3}\right| \leq \frac{1}{3}$$

P.23-24

1. Find the principal argument $\mathrm{Arg}(z)$ when
(a) $z = \dfrac{-2}{1+\sqrt{3}i}$; (b) $z = (\sqrt{3} - i)^6$.

2. Show that (a) $|e^{i\theta}| = 1$; (b) $\overline{e^{i\theta}} = e^{-i\theta}$.

4. Using the fact that the modulus $|e^{i\theta} - 1|$ is the distance between the point $e^{i\theta}$ and 1 (see Sec. 4). give a geometric argument to find a value of θ in the interval $0 \le \theta < 2\pi$ that satisfies the equation $|e^{i\theta} - 1| = 2$.

5. By writing the individual factors on the left in exponential form, performing the needed operations, and finally changing back to rectangular coordinates. show that
(a) $i(1 - \sqrt{3}i)(\sqrt{3} + i) = 2(1 + \sqrt{3}i)$; (b) $5i/(2 + i) = 1 + 2i$; (c) $(\sqrt{3} + i)^6 = -64$;
(d) $(1 + \sqrt{3}i)^{-10} = 2^{-11}(-1 + \sqrt{3}i)$.

7. Let z be a nonzero complex number and n a negative integer ($n = -1, -2, ...$). Also, write $z = re^{i\theta}$ and $m = -n = 1, 2, ...$ Using the expressions

$$z^m = r^m e^{im\theta} \quad \text{and} \quad z^{-1} = \left(\frac{1}{r}e^{-i\theta}\right),$$

verify that $(z^m)^{-1} = (z^{-1})^m$ and hence that the definition $z^n = (z^{-1})^m$ in Sec. 7 could have been written alternatively as $z^n = (z^m)^{-1}$.

9. Establish the identity

$$1 + z + z^2 + ... + z^n = \frac{1 - z^{n+1}}{1 - z} \quad (z \neq 1)$$

and then use it to derive Lagrange's trigonometric identity:

$$1 + \cos\theta + \cos 2\theta + ... + \cos n\theta = \frac{1}{2} + \frac{\sin[(2n+1)\theta/2]}{2\sin(\theta/2)} \quad (0 < \theta < 2\pi)$$