MATH 3060 Mathematical Analysis III

Tutorial 1 (September 19)

The following were discussed in the tutorial this week:

1. Definition of a Fourier series and the notation $\sim$.

2. Partial sum of a Fourier series and the notion of convergence of a Fourier series.

3. Riemann-Lebesgue lemma.

4. Recall the definition of Lipschitz condition:

   Definition. *A function f on $[a, b]$ is said to satisfy a Lipschitz condition if there is $L > 0$ such that*
   $$|f(x) - f(y)| \leq L|x - y|, \quad \forall x, y \in [a, b].$$

5. Recall Theorem 1.7 in the note:

   Theorem. *Let f be a 2π-periodic function satisfying a Lipschitz condition. Then its Fourier series converges uniformly to f itself.*

6. Let f be a 2π-periodic function given by
   $$f(x) = |\cos x|, \quad x \in \mathbb{R}.$$
   Compute the Fourier series of f. Show that
   $$|\cos x| = \frac{2}{\pi} + \frac{4}{\pi}\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{4m^2 - 1} \cos(2mx), \quad \forall x \in \mathbb{R},$$
   and the convergence is uniform. (**Hint:** Show that f satisfies
   $$|f(x) - f(y)| \leq |x - y|, \quad \forall x, y \in \mathbb{R}.)$$

7. The following is left as an exercise:

   Let f be a 2π-periodic function which is Riemann integrable on $[-\pi, \pi]$. Show that
   $$\lim_{n\to\infty} \int_{-\pi}^{\pi} f(x)|\cos nx|dx = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx.$$
   (**Hint:** Use the result in 6 and Riemann-Lebesgue lemma.)