MATH3060 HW 3 Due date: Oct 5, 2016

1. Let f, g and $h \in R[a,b]$. Show that

$$\|f-g\|_2 \le \|f-h\|_2 + \|h-g\|_2 .$$

When does the equality sign hold?

2. Let $\{\varphi_k\}_{k=1}^{\infty}$ be an orthonormal set on $R[a,b]$.

Show that $\forall f \in R[a,b]$,

$$\sum_{k=1}^{\infty} \langle f, \varphi_k \rangle_2^2 \le \int_a^b f^2 .$$

(Note that $\{\varphi_k\}_{k=1}^{\infty}$ may not be a basis.)

3. Let f, g be 2π-periodic functions integrable on $[-\pi, \pi]$.

Show that

$$\int_{-\pi}^{\pi} fg = 2\pi\, a_0(f)\, a_0(g) + \pi \sum_{n=1}^{\infty} \left[a_n(f) a_n(g) + b_n(f) b_n(g) \right]$$

where a_0, a_n, b_n are corresponding Fourier coefficients.

4. Show that

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^4} = \frac{\pi^4}{96}$ by Fourier series of $|x|$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$ by Fourier series of x^2

5. Use Wirtinger's inequality to show that $\forall f \in C[0,\pi]$ satisfying $f(0)=f(\pi)=0$ & $f'(x)$ exists $\forall x \in [0,\pi]$ and $f' \in R[0,\pi]$, the inequality

$$\int_0^\pi |f|^2 \leq \int_0^\pi |f'|^2 \quad \text{holds.}$$

When does the equality sign hold?