MATH3060 HW 2 Due date: Sep 28, 2016

1. A finite Fourier series is of the form

$$a_0 + \sum_{n=1}^{N} (a_n \cos nx + b_n \sin nx).$$

A trigonometric polynomial is of the form

$$p(\cos x, \sin x)$$

where $p(x,y)$ is a polynomial of 2 variables x, y.
Show that a function is a trigonometric polynomial if and only if it is a finite Fourier series.

2. Let f be 2π-periodic function integrable on $[-\pi, \pi]$.
Show that $$F(x) = \int_0^x f(x)\,dx$$
is 2π-periodic if and only if $\int_{-\pi}^{\pi} f = 0$.
When this holds, find $a_n(F)$ & $b_n(F)$ in terms of $a_n(f)$ and $b_n(f)$. (May assume f cts in your soln.)

3. Let f be a C^∞ 2π-periodic cpx-valued function.
Show that the (cpx) Fourier coefficient $c_n = o(\frac{1}{n^k})$ as $n \to \pm\infty$ for every k.

4. Show that $\frac{\pi^2}{12} = \sum_{k=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

(Hint: consider Fourier expansion of x^2 on $[-\pi, \pi]$.)

5. Using Thm 1.5 of §1.3 (in the Notes of Lecture 3), show that for $t \in (0,1)$,

$$\frac{\pi \cos tx}{\sin t\pi} = \frac{1}{t} + \sum_{n=1}^{\infty} \frac{2t}{t^2 - n^2} (-1)^n \cos nx, \quad x \in [-\pi, \pi]$$