MATH3060 HW1 Due date: Sep 21, 2016

1. Let f be a 2π-periodic function integrable on $[-\pi, \pi]$. Show that it is integrable over any finite interval and

$$\int_I f(x)\,dx = \int_{-\pi}^{\pi} f(x)\,dx$$

for any interval of length 2π.

2. Show that the Fourier series of every even function is a cosine series and the Fourier series of every odd function is a sine series

3. Each of the following functions (on the left-hand side) are defined on $[-\pi, \pi]$. Sketch the 2π-periodic extension and verify the Fourier expansion on the right-hand side

(a) $$x^2 \sim \frac{\pi^2}{3} - 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$$

(b) $$|x| \sim \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(2n-1)x$$

(c) $$f(x) = \begin{cases} 1, & x \in [0, \pi] \\ -1, & x \in [-\pi, 0) \end{cases} \sim \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)x$$

(d) $$g(x)=\begin{cases} x(\pi-x), & x\in[0,\pi] \\ x(\pi+x), & x\in(-\pi,0) \end{cases} \sim \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}\sin(2n-1)x$$

(4) Consider the function $f(x)=x^2$ on $(0,2\pi]$ and its 2π-periodic extension $\tilde{f}(x)$ by

$$\tilde{f}(x)=f(x-2k\pi) \quad \text{for } x\in(2k\pi, 2(k+1)\pi], \ \forall k\in\mathbb{Z}.$$

Sketch $\tilde{f}$ and show that

$$\tilde{f}\sim\frac{4\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}-4\pi\sum_{n=1}^{\infty}\frac{\sin nx}{n}.$$