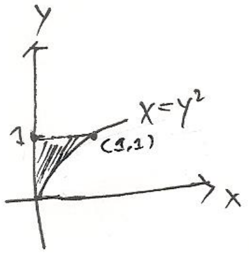
Ex 15-2

15a) $\displaystyle\int_0^{\ln 3}\int_{e^{-x}}^{1} dy\,dx$

b) $\displaystyle\int_{1/3}^{1}\int_{-\ln y}^{\ln 3} dx\,dy$

23.



$$\int_0^1\int_0^{y^2} 3y^3 e^{xy}\,dx\,dy$$

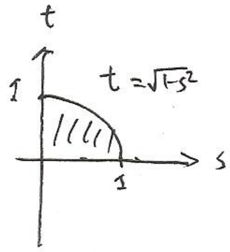
$$= \int_0^1 \left[3y^2 e^{xy}\right]_{x=0}^{x=y^2} dy$$

$$= \int_0^1 3y^2 e^{y^3} - 1 \; dy$$

$$= \left[e^{y^3} - y\right]_0^1$$

$$= e - 2$$

30.



$$\int_0^1\int_0^{\sqrt{1-s^2}} 8t\,dt\,ds$$

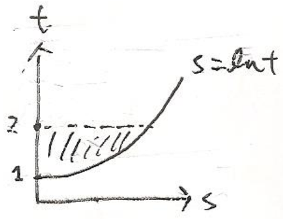
$$= \int_0^1 \left[4t^2\right]_{t=0}^{t=\sqrt{1-s^2}} ds$$

$$= \int_0^1 4 - 4s^2\; ds$$

$$= \left[4s - \frac{4}{3}s^3\right]_0^1$$

$$= \frac{8}{3}$$

28.



$$\int_1^2\int_0^{\ln t} e^s \ln t\; ds\,dt$$

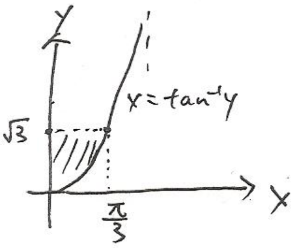
$$= \int_1^2 \ln t \left[e^s\right]_{s=0}^{s=\ln t} dt$$

$$= \int_1^2 t\ln t - \ln t\; dt.$$

$$= \left[\frac{1}{2}t^2\ln t - \frac{1}{4}t^2 - t\ln t + t\right]_1^2$$

$$= \frac{1}{4}.$$

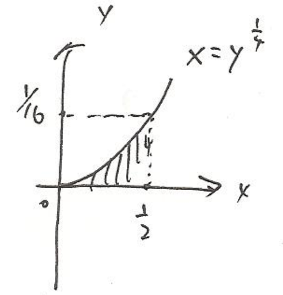
46.



$$\int_0^{\sqrt{3}}\int_0^{\tan^{-1} y} \sqrt{xy}\,dx\,dy$$

$$= \int_0^{\frac{\pi}{3}}\int_{\tan x}^{\sqrt{3}} \sqrt{xy}\,dy\,dx$$

53



$$\int_0^{1/16}\int_{y^{1/4}}^{1/2} \cos(16\pi x^5)\,dx\,dy$$

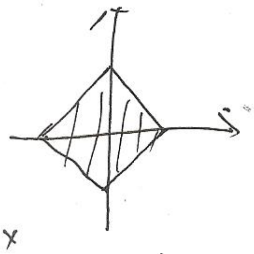
$$= \int_0^{1/2}\int_0^{x^4} \cos(16\pi x^5)\,dy\,dx$$

$$= \int_0^{1/2} x^4\cos(16\pi x^5)\,dx$$

$$= \left[\frac{1}{80\pi}\sin(16\pi x^5)\right]_0^{1/2}$$

$$= \frac{1}{80\pi}$$

64. Volume $= \int_{|x|+|y|\le 1} (3-3x)-(0)\,dA$



$$= \int_{-1}^{0}\int_{-x-1}^{x+1} 3-3x\,dy\,dx + \int_0^1\int_{x-1}^{1-x} 3-3x\,dy\,dx$$

$$= \int_{-1}^{0}(3-3x)(2x+2)\,dx + \int_0^1 (3-3x)(2-2x)\,dx$$

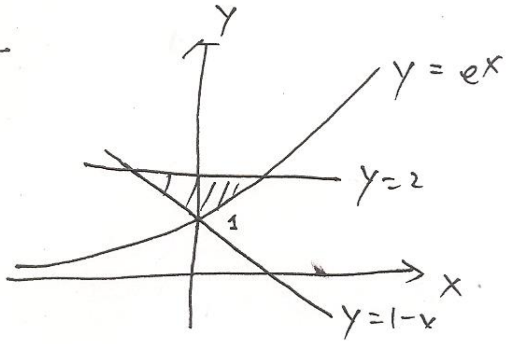
$$= 6\int_{-1}^{0} 1-x^2\,dx + 6\int_0^1 (1-x)^2\,dx$$

$$= 6\left[x-\tfrac{1}{3}x^3\right]_{-1}^{0} + 6\left[\tfrac{1}{3}(x-1)^3\right]_0^1$$

$$= 4 + 2 = 6.$$

15.3

10.



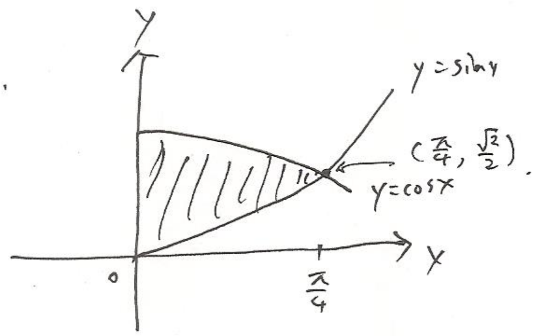
$$\text{Area} = \int_1^2 \int_{1-y}^{\ln y} dx\,dy$$

$$= \int_1^2 \ln y - 1 + y \, dy$$

$$= \left[y\ln y - 2y + \frac{y^2}{2} \right]_1^2$$

$$= 2\ln 2 - \frac{1}{2}.$$

15.



$$\text{Area} = \int_0^{\frac{\pi}{4}} \int_{\sin x}^{\cos x} dy\,dx$$

$$= \int_0^{\frac{\pi}{4}} \cos x - \sin x \, dx$$

$$= \left[\sin x + \cos x \right]_0^{\frac{\pi}{4}}$$

$$= \sqrt{2} - 1$$

20.

Over the square:

~~Avg~~ Average of Integral $= \frac{1}{1^2}\int_0^1 \int_0^1 xy \, dx\,dy$

$$= \int_0^1 x\,dx \int_0^1 y\,dy$$

$$= 1 \cdot 1 = 1.$$

Over the quarter circle:

Average of Integral $= \frac{1}{\frac{\pi}{4}} \int_0^1 \int_0^{\sqrt{1-x^2}} xy \, dy\,dx$

$$= \frac{4}{\pi} \int_0^1 x\left(\frac{1}{2}(1-x^2)\right)dx$$

$$= \frac{2}{\pi} \int_0^1 x - x^3 \, dx$$

$$= \frac{2}{\pi} \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1$$

$$= \frac{1}{2\pi} < 1.$$

So the average over the square is larger.