Ex 15.1 If you find any problems with the solution. please email

kfchan@math.cuhk.edu.hk.

8. $\int_1^4 \int_0^4 \left(\frac{x}{2} + \sqrt{y}\right) dx\,dy$

$= \int_1^4 \left[\frac{x^2}{4} + \sqrt{y}\cdot x\right]_{x=0}^{x=4} dy$

$= \int_1^4 \left(4 + 4\sqrt{y}\right) dy$

$= \left[4y + \frac{8}{3} y^{\frac{3}{2}}\right]_1^4$

$= \frac{92}{3}$ □

---

16. $\iint_R y \sin(x+y)\, dA$, $R: -\pi \le x \le 0$, $0 \le y \le \pi$

$= \int_0^{\pi} \int_{-\pi}^{0} y \sin(x+y)\, dx\, dy$

$= \int_0^{\pi} \left[-y\cos(x+y)\right]_{x=-\pi}^{x=0} dy$

$= \int_0^{\pi} \left(-y\cos(y) + y\cos(y-\pi)\right) dy$

$= -2\int_0^{\pi} y\cos y\, dy$

$= -2\int_0^{\pi} y\, d\sin y$

$= \left[-2y\sin y\right]_0^{\pi} + 2\int_0^{\pi} \sin y\, dy$

$= \left[-2y\sin y\right]_0^{\pi} + \left[-2\cos y\right]_0^{\pi}$

$= 0 + 4 = 4$ □

22.

$$\int_0^1 \int_0^{\pi} y\cos xy \, dx\, dy$$

$$= \int_0^1 \left[\sin xy \right]_{x=0}^{x=\pi} dy$$

$$= \int_0^1 \sin \pi y \, dy$$

$$= \left[-\frac{1}{\pi} \cos \pi y \right]_0^1$$

$$= \frac{2}{\pi} \quad \square$$

26

$$\text{Volume} = \int_0^2 \int_0^4 \frac{y}{2} \, dx \, dy$$

$$= \int_0^2 2y \, dy$$

$$= \left[y^2 \right]_0^2$$

$$= 4 \quad \square$$

28 Volume $= \int_0^2 \int_0^1 4-y^2 \, dx\,dy$

$= \int_0^2 4-y^2 \, dy$

$= \left[4y - \frac{1}{3}y^3\right]_0^2$

$= \frac{16}{3}$ □