MATH3060 HW 8 Due date: Dec 3, 2020 (at 12:00 noon)

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(1) Show that the set $\{ \frac{k}{2^l} \in \mathbb{R} : k, l \in \mathbb{Z} \}$ is of 1st category, but not nowhere dense.

(2) Show that $\mathcal{C} = \{ f \in C[0,1] : \int_0^1 f(x)\,dx \neq 0 \}$ is a residual set in $(C[0,1], d_\infty)$.

(3) Show that $\mathcal{P} = \{ P \in C[0,1] : p \text{ is a polynomial} \}$ is a set of 1st category.

(4) Show that a nonempty countable metric space with no isolated point cannot be complete.

(5) Let $l_2 = \{ \{x_n\}_{n=1}^\infty : \sum_{n=1}^\infty x_n^2 < \infty \}$ with metric

$$d_2(\{x_n\}, \{y_n\}) = \sqrt{\sum_{n=1}^\infty (x_n - y_n)^2}.$$

Show that $H = \{ \{x_n\}_{n=1}^\infty \in l_2 : |x_n| \leq \frac{1}{n} \}$ is nowhere dense in (l_2, d_2).

(End)