MATH3060 HW6 Due date: Nov 13, 2020 (at 12:00 noon)

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(1) Let $f:\mathbb{R}\to\mathbb{R}$ be C^2 and $f(x_0)=0$, $f'(x_0)\neq 0$. Show that there exists $\rho>0$ such that

$$Tx = x - \frac{f(x)}{f'(x)}, \quad x\in(x_0-\rho, x_0+\rho)$$

is a contraction. (This is the Newton's method.)

(2) Consider the function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x)=\begin{cases}\frac{1}{2}x + x^2\sin\frac{1}{x}, & x\neq 0\\ 0, & x=0.\end{cases}$$

Show that f is differentiable at $x=0$ with $f'(0)=\frac{1}{2}$, but it has no local inverse at $x=0$. Does it contradict the inverse function theorem?

(Con't on next page)

(3) Let $a>0$, define a mapping $T: C[-a,a] \to C[-a,a]$ by

$$Tx(t) = 1 + \int_0^t x(s)\,ds.$$

Let $x(t) \equiv 1$ on $[-a,a]$

Find $T^n x$, $\forall n \geq 0$. Does $\{T^n x\}$ converge in $(C[-a,a], d_\infty)$? If so, what is the limit?

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(End)