Periodic orbits of discretized rotations

Shigeki Akiyama, Tsukuba University

Discretized rotations are special kind of domain exchange transformations. In general, they have zero entropy. It is well known that for some special rational angles, they contain self-inducing structures, but we know little on the general cases. Here we show that for any discretized rotation, there must exist infinitely many periodic orbits. We use lattice point counting technique in number theory to obtain the result. This seems to be the first non-trivial general statement on the orbits of these systems.

This is join work with Attila Petho.

Hanoi attractors and the Sierpinski gasket: Geometric and analytic convergence

Patricia Alonso-Ruiz, University of Siegen

For each $\alpha \in (0, 1/3)$, we define the so-called Hanoi attractor K_α. We show that the sequence $(K_\alpha)_\alpha$ converges to the Sierpinski gasket K with respect to the Hausdorff metric as α tends to 0, and the Hausdorff dimension converges too. In view of this result, we look for an analytic convergence: We define a Laplacian on K_α by means of a suitable Dirichlet form. We study its asymptotic behavior and compare it with the one for the Laplacian on K. Since K_α is not self-similar, we have to modify the classical construction of the Laplace operator for p.c.f. self-similar fractals by Kigami, Strichartz et al.

Eigenvalues of the Laplacian on Cantor-sets via modified trigonometric functions

Peter Arzt, University of Siegen

We investigate the operator $-\frac{d}{d\mu}\frac{d}{dx}$ acting on $L_2([0,1],\mu)$ where μ is a self-similar measure with support K. This operator we call Laplacian on K. We define analogs of the sine and cosine function to describe the eigenfunctions of $-\frac{d}{d\mu}\frac{d}{dx}$. Further modified sine functions are defined as power series such that their zero points squared are the Neumann- and Dirichlet-eigenvalues, respectively. In order to determine the behavior of the eigenvalues we derive properties of those 'modified trigonometric functions'.

Patterns generation problems arising in multiplicative integer systems

Jung-Chao Ban, National Dong Hwa University

This talk considers a multiplicative integer system using a method that was developed for studying pattern generation problems. The entropy and the Minkowski dimensions of general multiplicative systems can thus be computed. A multi-dimensional decoupled system is investigated in three main steps. (I). identify the admissible lattices of the system; (II). compute the density of copies of admissible lattices of the same length; (III). Compute the number of admissible patterns on the admissible lattices. A coupled system can be decoupled by removing the multiplicative relation set and then performing procedures similar to those applied to a decoupled system. The admissible lattices are chosen to be maximum graphs of different degrees which are independent of each other. The entropy can be obtained after the remaining error term is shown to approach zero as the degree of the admissible lattice tends to infinity.

Bernoulli convolutions and branching dynamical systems

Christoph Bandt, Greifswald, Germany

We present a new approach to Bernoulli convolutions which uses the inverse maps of the iterated functions system. Let $J_1, ..., J_m$ be subsets of a set J, and $g_1, ..., g_m$ mappings from J_i into J. The mapping G defined by

$$G(x) = \{g_i(x) \,|\, x \in J_i\} \quad \text{for } x \in J$$

and $G(A) = \bigcup_{a \in A} G(a)$ for finite subsets A of J is called a branching dynamical system on J. The elements of the set $G^n(x)$ are called successors of x in generation $n = 1, 2, ...$ If the limit of $\sqrt[n]{|G^n(x)|}$ exists, it is called the growth factor of successors of x. We are also interested in the asymptotic distribution of successors of a point and in invariant measures of G.

This concept, applied to the inverse maps of the contractions which generate a Bernoulli convolution, will provide interesting results on the Erdös problem, as well as more general open questions.

Analytic continuation of fractal functions

Michael Barnsley, Australian National University

An important class F of continuous functions $f : [0, 1] \to \mathbb{R}$ can be represented as attractors of analytic iterated function systems (IFSs), (i.e. IFSs that comprise a finite number of real analytic functions). We have: (a) F includes all real analytic functions on [0,1]; (b) F includes segments of wavelets; (c) F includes fractal interpolation functions. We prove (a), comment on (b) and (c), and introduce a generalization of analytic continuation that applies to all functions in F. We outline the proof that, under general conditions, the set of continuations is independent of the IFS.

This is joint work with Andrew Vince (UFL).

Recent advances in Mandelbrot martingales theory

Julien Barral, Université Paris 13

We will give an overview of recent progress in the renormalization of partition functions of energy models related to the branching random walk, and how this can be used to complete in several directions the theory of limit of (possibly normalized) Mandelbrot martingales.

Sets of exact approximation order by rational numbers

Yann Bugeaud, Université de Strasbourg

For a function $\Psi : \mathbf{R}_{>0} \to \mathbf{R}_{>0}$, let

$$\mathcal{K}(\Psi) := \left\{ \xi \in \mathbf{R} : \left| \xi - \frac{p}{q} \right| < \Psi(q) \text{ for infinitely many rational numbers } \frac{p}{q} \right\}$$

denote the set of Ψ-approximable real numbers and let

$$\text{Exact}(\Psi) := \mathcal{K}(\Psi) \setminus \bigcup_{m \geq 2} \mathcal{K}\big((1 - 1/m)\Psi\big)$$

be the set of real numbers approximable to order Ψ and to no better order. In other words, Exact(Ψ) is the set of real numbers ξ such that

$$|\xi - p/q| < \Psi(q) \qquad \text{infinitely often}$$

and

$$|\xi - p/q| \geq c\,\Psi(q) \qquad \text{for any } c < 1 \text{ and any } q \geq q_0(c, \xi),$$

where $q_0(c, \xi)$ denotes a positive real number depending only on c and on ξ. When Ψ is non-increasing and satisfies $\Psi(x) = o(x^{-2})$, we establish that Exact(Ψ) has Hausdorff dimension $2/\lambda$, where λ is the lower order at infinity of the function $1/\Psi$. This refines a classical result of Jarník and Besicovitch, who, independently, established some eighty years ago that $\mathcal{K}(\Psi)$ has Hausdorff dimension $2/\lambda$.

Furthermore, we study the set Exact(Ψ) when Ψ is not assumed to be non-increasing and show that the set Exact(Ψ) is uncountable for a large class of functions Ψ satisfying $\Psi(x) = o(x^{-2})$.

This is partially a joint work with C. G. Moreira.

The dimension of the full nonuniformly hyperbolic horseshoe

Yongluo Cao, Suzhou University

In this talk, we will consider the dimension and dynamics of the full nonuniformly hyperbolic horseshoe. We study the dynamics and dimension of the full nonuniformly hyperbolic horseshoe by constructing an induced map which is hyperbolic.

Periodic billiard orbits in planar Sierpinski carpets

Joe Po-Chou Chen, Cornell University

In this talk I will present results, joint with Robert Niemeyer, on the identification of periodic billiard orbits in the self-similar Sierpinski carpet S_a, where a is an odd integer ≥ 3, and S_a is constructed in the first stage by removing from the unit square $[0,1]^2$ a square of side a^{-1} centered at $(\frac{1}{2}, \frac{1}{2})$. (So S_3 is the standard Sierpinski carpet.) Our results elaborate upon the recent work of E. Durand-Cartegena and J. Tyson, where they showed that in each S_a, there are only a finite number of rational slopes that a nontrivial rectifiable curve can take on. We provide the necessary conditions where such a curve, upon being reflected and unfolded onto the pre-fractal approximations of S_a, gives rise to a sequence of compatible periodic orbits which becomes eventually constant. Explicit examples are given.

Reflecting random walk in fractal domains

Zhen-Qing Chen, University of Washington

In this talk, we show that for any bounded Euclidean domain D, simple random talk on "maximal connected" subsets of grids $(2^{-d}\mathbb{Z}^d) \cap D$ whose filled-in interiors are inside D converges weakly to reflected Brownian motion on D. Tools from Dirichlet form theory play an important role in our approach.

This is based on a joint work with K. Burdzy.

A new approach to Gaussian upper bounds for the heat kernel on doubling metric measure spaces

Thierry Coulhon, The Australian National University

On metric measure spaces endowed with a Dirichlet form and satisfying the volume doubling property, we show some characterizations of the Gaussian upper bound of the heat kernel in terms of one-parameter weighted inequalities which correspond respectively to the Nash inequality and to a Gagliardo-Nirenberg type inequality when the volume growth is polynomial. We relate these inequalities to the local Nash inequalities introduced by Kigami. We also treat the case of subgaussian heat kernel upper bounds which are typical of fractals.

This is joint work with Salahaddine Boutayeb and Adam Sikora.

Probabilistic Approach to Radix Representation and Pseudodigits

Eva Curry, Acadia University

Given an $n \times n$ dilation matrix A and complete digit set D, we can define a Markov process $\{X_n\}$ on $\mathbb{Z}^n$ by setting $X_0 = x$ for some arbitrarily chosen point x and $X_n = A(x + d_n)$, where $d_n \in D$ is chosen randomly with a uniform distribution. We'll look at some features of this process. Reversing this process gives us the mapping $U(x) = A^{-1}(x+d_n)$, where d_n is the digit representing the coset of $\mathbb{Z}^n/A\mathbb{Z}^n$ containing x. (This is the standard Euclidean algorithm.) When the pair (A, D) give a radix representation for all of $\mathbb{Z}^n$, all orbits of $U(x)$ reach 0. Otherwise, there are nonzero periodic orbits (a representative of which called be chosen as a "pseudodigit"). We'll discuss what is known or conjectured about the structure of these nonzero periodic orbits.

Spectra on fractal measures

Xinrong Dai, Sun Yat-sen University

In this talk, we will discuss some spectra properties associated with some fractal measures. In particular, we will consider spectral property of the Bernoulli convolution and the Cantor type measure, the tree structure of the maximal orthogonal set and spectrum, and the sparsity of the spectrum.

An analytic inequality and higher multifractal moments

Kenneth J. Falconer, University of St Andrews

The talk will present an inequality relating to multipotential integrals on tree-like structures. Several applications to generalized dimensions will be discussed, including to almost self-affine measures, to images of measures under Brownian-type processes, and to higher moments of random cascade measures.

Multifractal analysis of some multiple ergodic average – The Hausdorff spectrum

Ai Hua Fan, Université de Picardie Jules Verne

The multiple ergodic theory was initiated by Fürstenberg. We would like to study multiple ergodic averages from multifractal analysis point of view. This question was raised by Fan, Liao and Ma who obtained first results in some special cases. In this talk, we discuss the multiple ergodic averages

$$\frac{1}{n}\sum_{k=1}^{n}\varphi(x_k, x_{kq}, \cdots, x_{kq^{\ell-1}})$$

on the symbolic space $\Sigma_m = \{0, 1, \cdots, m-1\}^{\mathbb{N}^*}$ where $m \geq 2, \ell \geq 2, q \geq 2$ are integers. In this case, a complete solution to the problem of multifractal analysis of the limit of the multiple ergodic averages is obtained by Fan, Schmeling and Wu (this is a part of Meng WU's thesis). A decisive step for the resolution is inspired by Kenyon, Peres, Solomyak's introduction of what we call telescopic product measures. All these measures are proved to be exact and some of them play the Gibbs measures' role in our study of multiple ergodic averages.

In his talk, J. Schmeling will discuss the invariant part of the multifractal level sets. Remarkable differences between the new theory and the classical one ($\ell = 1$) will be shown.

Our studied multiple ergodic averages are defined on full shift space by function φ depending only on the first coordinates. We point out that there are still challanging problems on the subject. For example, how about subshift of finite type or more general dynamics? what happens when φ depend on several coordinates?

On the strongly tridiagonal competitive-cooperative system

Chun Fang, University of Helsinki

In this talk, we study the properties and structures of persistence solutions of a tridiagonal competitive-cooperative system. More precisely, we show that any hyperbolic ω-limit set of a strongly tridiagonal competitive-cooperative system is 1-cover of its base flow and similar results also hold under perturbation. We also consider the ergodic properties of the strongly competitive-cooperative system.

Modified singular value functions and self-affine carpets

Jonathan Fraser, University of St Andrews

We will discuss the dimension theory of certain classes of self-affine subsets of the plane. Since the introduction of the Bedford-McMullen carpets in the mid 80s, the study of 'exceptional classes' of self-affine sets has attracted a great deal of attention in the literature with particular interest being in computing the dimensions of increasingly more general classes of set. Important examples have been introduced by Gatzouras and Lalley ('92), Feng and Wang ('05) and Barański ('07). These constructions are sometimes referred to as 'exceptional' as they often provide exceptions to Falconer's seminal result on the almost sure dimensions of self-affine sets based on singular value functions. Here we introduce a new class of self-affine sets which is more general than the four aforementioned classes and we compute the packing and box dimensions via a modified singular value function, hence shedding some light on the interplay between the exceptional constructions and Falconer's almost sure formula.

Spectral analysis of V-variable Sierpinski gaskets

Uta Freiberg, University of Siegen

Self similar fractals are often used in modeling porous materials. However, the assumption of strict self similarity could be too restricting. So, we present several models of random fractals which could be used instead. After recalling the classical approaches of random homogenous and recursive random fractals, we show how to interpolate between these two models with the help of so called V-variable fractals. This concept (developed by Barnsley, Hutchinson and Stenflo) allows the definition of new families of random fractals, hereby the parameter V describes the degree of "variability" of the realizations. We discuss how the degree of variability influences the spectral asymptotics of corresponding Dirichlet forms. Moreover, on–diagonal heat kernel estimates are presented.

Measure of Self-Affine Sets and Associated Densities

Jean-Pierre Gabardo, McMaster University

Let B be an $n \times n$ real expanding matrix and $\mathcal{D}$ be a finite subset of $\mathbb{R}^n$ with $0 \in \mathcal{D}$. The self-affine set $K = K(B, \mathcal{D})$ is the unique compact set satisfying the set equation $BK = \bigcup_{d \in \mathcal{D}} (K + d)$. In the case where $\mathrm{card}(\mathcal{D}) = |\det B|$, we relate the Lebesgue measure of K to the upper Beurling density of the associated measure $\mu = \lim_{s \to \infty} \sum_{\ell_0, \ldots, \ell_{s-1} \in \mathcal{D}} \delta_{\ell_0 + B\ell_1 + \cdots + B^{s-1}\ell_{s-1}}$. If, on the other hand, $\mathrm{card}(\mathcal{D}) < |\det B|$ and B is a similarity matrix, we relate the Hausdorff measure $\mathcal{H}^s(K)$, where s is the similarity dimension of K, to a corresponding notion of upper density for the measure μ.

This is joint work with Xiaoye Fu.

Iteration of polynomials, functional equations, and fractal zeta functions

Peter Grabner, Technische Universität Graz

Iterative functional equations, such as the Poincaré equation and the Böttcher equation have been studied in complex dynamics in order to obtain a deeper understanding of the local behavior of the iterates of polynomials $P(z)$. The Poincaré functional equation

$$f(\lambda z) = P(f(z)), \quad P'(f(0)) = \lambda$$

for $f(0)$ a fixed point of P provides a local linearisation of the function P around $f(0)$, if $|\lambda| > 1$. Similarly, the Böttcher equation

$$g(z)^d = g(P(z)), \quad d = \deg P$$

provides a normalization of P around infinity. Combining these two functions provides precise information about the asymptotic behavior of $f(z)$ for $z \to \infty$ and $f(z) \to \infty$ in some angular region.

Further interest in Poincaré functions f comes from the fact that the spectrum of the Laplace operator on fractals with spectral decimation can be described in terms of level sets of f. This allows to give an analytic continuation of the ζ-function of this Laplace operator to the whole complex plane. Furthermore, special values of this ζ-function can be computed. This allows to define and compute the Casimir energy on fractals like the Sierpiński gasket and its higher dimensional analogues.

This talk is based on joint work with Gregory Derfel (Ben Gurion University, Beer Sheva) and Fritz Vogl (Vienna University of Technology).

On stochastic completeness of jump processes

Alexander Grigor'yan, Universität Bielefeld

We discuss conditions for stochastic completeness of symmetric jump processes on metric measure spaces. Assuming that the distance function is adapted in a certain sense to the jump kernel, a sufficient condition for stochastic completeness can be obtained in terms of the volume function of balls. We use this result to obtain the following criterion for stochastic completeness of a continuous time random walk on a graph with a counting measure: if the volume growth with respect to the graph distance is at most cubic then the random walk is stochastically complete, where the cubic volume growth is sharp.

Frames for fractal measures and group representations

Deguang Han, University of Central Florida

I will discuss some recent results on frames of exponentials and for group representations that are motivated by the Feichtinger Frame Conjecture. The talk will be focused on the existence problem of exponential frames (or more generally, frame measures) for fractal measures, and the frame conjecture for group representation frames.

Exponential spectra in $L^2(\mu)$

Xinggang He, Central China Normal University

Abstract: Let μ be a Borel probability measure with compact support. We consider exponential type orthonormal bases, Riesz bases and frames in $L^2(\mu)$. We show that if $L^2(\mu)$ admits an exponential frame, then μ must be of pure type. We also classify various μ that admits either kind of exponential bases, in particular, the discrete measures and their connection with integer tiles. By using this and convolution, we construct a class of singularly continuous measures that has an exponential Riesz basis but no exponential orthonormal basis. It is the first of such kind of examples.

This is joint work with Chun-kit Lai and Ka-Sing Lau.

Geodesic distances and intrinsic distances on some fractal sets

Masanori Hino, Kyoto University

The off-diagonal Gaussian asymptotics of the heat kernel density associated with a strong local Dirichlet form is often described by using the intrinsic distance (or the Carnot–Caratheodory distance; cf. [5, 4] and the references therein). When the underlying space has a Riemannian structure, the geodesic distance is defined as well, and it coincides with the intrinsic distance in good situations.

Then, what if the underlying space is a fractal set? In typical examples, the heat kernel asymptotics is *sub*-Gaussian; accordingly, the intrinsic distance vanishes identically. However, if we take (a sum of) energy measures as the underlying measure, we can define the nontrivial intrinsic distance as well as the geodesic distance, and can pose a problem whether they are identical. For the 2-dimensional standard Sierpinski gasket, the affirmative answer has been obtained ([2, 3]) by using detailed information on the transition density with probabilistic arguments. In this talk, I will discuss this problem in a more general framework and provide some partial answers based on purely analytic arguments.

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Energy Measures of Harmonic Functions on the Sierpiński Gasket

Ching Wei Ho, The Chinese University of Hong Kong

We study energy measures on SG based on harmonic functions. We characterize the positive energy measures through studying the bounds of Radon-Nikodym derivatives with respect to the Kusuoka measure. We prove a limited continuity of the derivative on the graph V_* and express the average value of the derivative on a whole cell as a weighted average of the values on the boundary vertices. We also prove some characterizations and properties of the weights.

This is a joint work with Renee Bell and Robert S. Strichartz.

Progress on self similar sets and measures with overlaps

Mike Hochman, The Hebrew University

Let $X \subseteq \mathbb{R}$ be a self-similar set with similarity dimension s. Conjecturally, if the dimension of X is "less than expected" – that it, if $\dim X < \min\{1, s\}$ – then X has exact overlaps. I will discuss a recent result saying that if the above inequality holds, then there are super-exponentially close cylinder sets. A more analytical form of this theorem holds for self-similar measures. As applications, we prove that Bernoulli convolutions have full dimension outside a set of parameters of dimension 0, and that if $X \subset \mathbb{R}$ is the middle-α Cantor set then $\dim(X + tX) = \min\{1, 2\dim X\}$ for all t outside a 0-dimensional set of parameters t.

Heat kernels and Green functions on metric measure spaces

Jiaxin Hu, Tsinghua University

We give a new equivalence for the heat kernel estimate of a local Dirichlet form. Namely, a two-sided estimate of the heat kernel is equivalent to the conjunction of the volume doubling property, the elliptic Harnack inequality and a certain estimate of the capacity between concentric balls.

This is joint work with Alexander Grigor'yan.

Stable sets in $\mathbb{Z}^n$-systems with positive entropy

Wen Huang, University of Science and Technology of China

In this talk, the chaoticity appearing in the $\mathbb{Z}_+^n$-stable sets of a $\mathbb{Z}^N$-dynamical system with positive entropy is investigated. It is shown that in any positive entropy $\mathbb{Z}^n$-system, there is a measure-theoretically rather big set such that the $\mathbb{Z}^n$-stable set of any point from the set contains a Mycielski Li-Yorke chaotical set under $\mathbb{Z}_-^n$.

Heat kernel estimates on a connected sum along a joint with a capacity growth

Satoshi Ishiwata, Yamagata University

In 2009, Grigor'yan and Saloff-Coste [2] proved sharp heat kernel estimates on connected sums with compact joint taking into account of a bottleneck effect. Recently, as a general result of them, Grigor'yan and the speaker obtained sharp heat kernel estimates on a connected sum of two copies of $\mathbb{R}^n$ along a surface of revolution in [1]. In this talk, we discuss the long time heat kernel estimates on a connected sum of two manifolds with two-sided heat kernel Gaussian estimates along a joint with a capacity growth.

This is joint work with Alexander Grigor'yan.

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Multifractal analysis of arithmetic functions

Stéphane Jaffard, Université Paris Est

One of the purposes of the multifractal analysis of functions is to determine their multifractal spectrums $d_f(H)$, which denotes the Hausdorff dimensions of the sets of points E_H where a function f has a given pointwise regularity exponent taking the value H.

Several categories of functions related with number theory proved to be multifractal, and their multifractal spectrums have been partly or completely determined. These include several examples of trigonometric series, one or several variables Davenport series, or the Minkowski function. The characterization of the corresponding fractal sets E_H usually involve Diophantine approximation properties.

We will review results in this area, and in particular, compare wavelet methods vs. direct ones for the determination of pointwise regularity; we will also present several open problems and show how they are (sometimes) related with conjectures in number theory.

Dimensions of random affine code tree fractals

Esa Järvenpää, University of Oulu

We study the dimension of code tree fractals, a class of fractals generated by a set of iterated function systems. We first consider deterministic affine code tree fractals, extending to the code tree fractal setting the classical result of Falconer and Solomyak on the Hausdorff dimension of self-affine fractals generated by a single iterated function system. We then calculate the almost sure Hausdorff, packing and box counting dimensions of a general class of random affine planar code tree fractals. The set of probability measures describing the randomness includes natural measures in random V-variable and homogeneous Markov constructions.

This is joint work with Maarit Järvenpää, Antti Käenmäki, Henna Koivusalo, Örjan Stenflo and Ville Suomala.

Hausdorff dimension of affine random covering sets in torus

Maarit Järvenpää, University of Oulu

We calculate the almost sure Hausdorff dimension of the random covering set $\limsup_{n\to\infty}(g_n + \xi_n)$ in d-dimensional torus $\mathbb{T}^d$, where the sets $g_n \subset \mathbb{T}^d$ are linear images of a set with nonempty interior and $\xi_n \in \mathbb{T}^d$ are independent and uniformly distributed random points. The dimension formula, derived from the singular values of the linear mappings, holds provided that the sequences of singular values are decreasing.

This is joint work with Esa Järvenpää, Henna Koivusalo, Bing Li and Ville Suomala.

On exact scaling log-Infinitely divisible cascades

Xiong Jin, University of St Andrews

I will talk about the extension of some classical results valid for canonical multiplicative cascades to exact scaling log-infinitely divisible cascades, that includes the extension of Kahane's results on the non-degeneracy and the finiteness of moments of positive orders, and the extension of Kahane's and Guivarc'h's results regarding the asymptotic behavior of the right-tail of the total mass.

This is joint work with Julien Barral.

Fundamental groups of Rauzy fractals

Timo Jolivet, Université Paris Diderot

Rauzy fractals are planar sets with fractal boundary that arise naturally in the study of substitutive dynamical systems. These fractals are compact and they are the closure of their interior, but their topological properties are otherwise very rich and diverse. For instance, they are not necessarily connected and their fundamental group can be uncountable. Many such properties can be algorithmically decided (Siegel-Thuswaldner 2009).

Motivations and results. All the currently known examples of nontrivial Rauzy fractal fundamental groups are uncountable, and up to now there seemed to be a dichotomy between the trivial group and very complicated uncountable fundamental groups.

We provide examples for which the situtation is intermediate, where the fundamental group is nontrivial and countable (with fundamental groups F_1 and F_6). Moreover, we prove that a nontrivial countable Rauzy fractal fundamental group is always isomorphic to the free group F_k on k elements, where k can be computed explicitly from σ in some cases.

We also work towards giving a full description of the fundamental group for some particular examples where it is uncountable, as has already been achieved for the Hawaiian Earring (Cannon-Conner 2000) or the Sierpiński gasket (Akiyama et al. 2009).

This is joint work with Benoît Loridant and Jun Luo.

Harmonic analysis of affine fractals

Palle E.T. Jorgensen, University of Iowa

In the talk we outline how an harmonic analysis for domains D in R^n behaves under scaling. By harmonic analysis we mean the possibility of a Fourier series representation by complex Fourier frequencies. The existence of such a representation is a rather restricting condition. By affine fractal we refer to iterated function systems from a prescribed fixed system of affine mappings. The scaling in the theory takes place with iterated powers of an expansive n by n matrices. We further study an interplay between the theory for $L^2(D)$ with respect to n-Lebesgue measure on the one hand; and $L^2(\mu)$ where μ is a related fractal measure, on the other. Positive powers of the scaling matrix generates possible spectra for $L^2(\mu)$ while negative powers of an affine system generates the fractal μ in the small.

The results presented will be joint work with S. Pedersen, and with D. Dutkay.

Multifractal analysis on infinitely generated self-affine sets

Antti Käenmäki, University of Jyväskylä

We calculate the Hausdorff dimension of a typical infinitely generated self-affine set. We also examine the multifractal analysis of Birkhoff averages in this setting.

This is joint work with H. Reeve.

Periodic and non-periodic aspects of the heat kernel asymptotics on Sierpiński carpets

Naotaka Kajino, Universität Bielefeld

The purpose of this talk is to present the author's recent results in [2, 3] on various short time asymptotics of the canonical heat kernel on Sierpiński carpets.

Let K be a generalized Sierpiński carpet, which is a compact subset of the Euclidean space, and let $p_t(x, y)$ be the transition density of the Brownian motion on K (the canonical heat kernel). Then it is well-known that there exist $c_1, c_2 \in (0, \infty)$ and $d_{\mathrm{w}} \in (2, \infty)$ such that for any $x \in K$,

$$c_1 \leq t^{d_{\mathrm{f}}/d_{\mathrm{w}}} p_t(x, x) \leq c_2, \qquad t \in (0, 1],$$

where d_{f} is the Hausdorff dimension of K with respect to the Euclidean metric. d_{w} is called the *walk dimension of* K, which is defined through the *time scaling factor* τ for the Brownian motion on K. Then it is natural to ask how $t^{d_{\mathrm{f}}/d_{\mathrm{w}}} p_t(x, x)$ behaves as $t \downarrow 0$ and in particular whether the limit $\lim_{t\downarrow 0} t^{d_{\mathrm{f}}/d_{\mathrm{w}}} p_t(x, x)$ exists. In fact, this limit does *not* exist for "generic" $x \in K$, and more strongly we have the following theorem. Recall that $f : (0, \infty) \to (0, \infty)$ is said to *vary regularly at* 0 if and only if the limit $\lim_{t\downarrow 0} f(\alpha t)/f(t)$ exists in $(0, \infty)$ for any $\alpha \in (0, \infty)$.

Theorem 0.1 ([2, Theorem 5.11]). *There exist $c_3 \in (0,\infty)$ and a Borel subset N of K satisfying $\nu(N) = 0$ for any self-similar measure ν on K, such that for any $x \in K \setminus N$,*

$$p_{(\cdot)}(x,x) \text{ does \textbf{not} vary regularly at } 0, \tag{NRV}$$

$$\limsup_{t\downarrow 0} \left| t^{d_\mathrm{f}/d_\mathrm{w}} p_t(x,x) - G(-\log t) \right| \geq c_3 \qquad \text{for any periodic function } G : \mathbb{R} \to \mathbb{R}. \tag{NP}$$

On the other hand, the (spectral) partition function $\mathcal{Z}(t)$, which is the trace of the associated heat semigroup at time t, is expected to exhibit log-periodic behavior, since $\mathcal{Z}(t)$ can be written as $\mathcal{Z}(t) = \sum_{n\in\mathbb{N}} e^{-\lambda_n t}$ by using the eigenvalues $\{\lambda_n\}_{n\in\mathbb{N}}$ of the associated Laplacian on K and $\{\lambda_n\}_{n\in\mathbb{N}}$ should strongly reflect the self-similarity of the space. Indeed, we have the following log-periodic asymptotic expansion of $\mathcal{Z}$, which is essentially a refinement of [1, Theorem 4.1].

Theorem 0.2 ([3]). *Set $d_k := \dim_\mathrm{H}\bigl(K \cap ([0,1]^{d-k} \times \{0\}^k)\bigr)$ for $k \in \{0,\ldots,d\}$, where $\dim_\mathrm{H}$ denotes Hausdorff dimension with respect to the Euclidean metric* (note that $d_0 = d_\mathrm{f}$, $d_{d-1} = 1$ and $d_d = 0$)*. Then there exist $c_4 \in (0,\infty)$ and continuous $\log\tau$-periodic functions $G_k : \mathbb{R} \to \mathbb{R}$, $k \in \{0,\ldots,d\}$, with G_0, G_1 being $(0,\infty)$-valued, such that*

$$\mathcal{Z}(t) = \sum_{k=0}^{d} t^{-d_k/d_\mathrm{w}} G_k(-\log t) + O\Bigl(\exp\bigl(-c_4 t^{-\frac{1}{d_\mathrm{w}-1}}\bigr)\Bigr) \qquad \text{as } t \downarrow 0.$$

References

[1] B. M. Hambly, Asymptotics for functions associated with heat flow on the Sierpinski carpet, *Canad. J. Math.* **63** (2011), 153–180.

[2] N. Kajino, *Non-regularly varying and non-periodic oscillation of the on-diagonal heat kernels on self-similar fractals*, 2012, preprint.

[3] N. Kajino, *Asymptotic expansion of the spectral partition function on self-similar sets*, 2012, in preparation.

Hausdorff dimension of metric spaces and Lipschitz maps onto cubes

Tamás Keleti, Eötvös Loránd University

Which compact metric spaces X can be mapped onto a k-dimensional cube by a Lipschitz map? One can easily see that $\mathcal{H}^k(X) > 0$ is a necessary condition, where $\mathcal{H}^k$ denotes the k-dimensional Hausdorff measure. It has been known for long time that this condition is not sufficient. We show that the condition $\dim_H X > k$, which is just a bit stronger than the necessary condition $\mathcal{H}^k(X) > 0$, is already sufficient.

The proof uses Peano curves with optimal Hölder constants and a recent deep result of Mendel and Naor about ultrametric spaces. In the special case when X is in a Euclidean space, self-similar sets can be used instead of the Mendel-Naor theorem.

As an application we essentially answer a question of Urbański by showing that the transfinite Hausdorff dimension (introduced by him) of an analytic subset A of a complete separable metric space is $\lfloor \dim_H A \rfloor$ if $\dim_H A$ is finite but not an integer, $\dim_H A$ or $\dim_H A - 1$ if $\dim_H A$ is an integer and at least ω_0 if $\dim_H A = \infty$.

This is joint work with András Máthé (Warwick) and Ondřej Zindulka (Prague).

Partition, volume doubling property and quasisymmetry on metric-measure spaces

Jun Kigami, Kyoto University

In this talk, we will propose a notion of partitions of compact metric spaces by trees. Moreover, we give a definition of a metric which is adapted to a partition. Under a metric which is adapted to a partition, an equivalent condition for a measure being volume doubling with respect to the metric and an equivalent condition for a metric being quasisymmetric with respect to the original metric are studied.

Hausdorff dimension of uniformly random self-affine sets

Henna Koivusalo, University of Oulu

We calculate the almost sure Hausdorff dimension of uniformly random self-affine sets. In many of the previous works, when calculating the dimension of self-affine fractals the result is only obtained for almost all translations. However, in our setting the translation vectors are fixed and non-random. The randomness is introduced by choosing the linear parts of the affine mappings using a uniform distribution, independently at each step of the construction. The proof relies on verification of the *self-affine transversality condition* introduced by Jordan, Pollicott and Simon in 2006.

At the time of the submission of this abstract the work is still in progress.

Fluctuations of recentered maxima of discrete Gaussian Free Fields on a class of recurrent graphs

Takashi Kumagai, Kyoto University

We provide conditions that ensure that the recentered maximum of the Gaussian free field on a sequence of graphs fluctuates at the same order as the field at the point of maximal variance. In particular, on a sequence of such graphs the recentered maximum is not tight, similarly to the situation in $\mathbb{Z}$ but in contrast with the situation in $\mathbb{Z}^2$. We show that our conditions cover a large class of “fractal” graphs.

This is joint work with O. Zeitouni.

Fourier frames on measure spaces

Chun Kit Lai, McMaster University

Fourier frame is a natural generalization of exponential orthonormal basis on $L^2(\mu)$. Motivated from the discovery of exponential orthonormal basis on the one-fourth Cantor measure, but not for the one-third one, there has been interest in understanding the kind of measures that admit some Fourier frames. By decomposing the measure into discrete, singular and absolutely continuous part, we show that measures with Fourier frames are of pure type and then we will report on the progress in Fourier frames by studying the types one by one.

Fractal properties of the Schramm-Loewner evolution

Gregory F. Lawler, University of Chicago

The Schramm-Loewner evolution (SLE_κ) is a an example of a random fractal curve in two-dimensions that arises in the scaling limit of models in statistical physics. I will discuss work with a number of co-authors about the fractal properties of the curve. In particular, I will discuss a recent result with M. Rezaei establishing that the "natural parametrization" of the curve is given by its d-dimensional Minkowski content. Here $d = d(\kappa)$ is the fractal dimension of the curves.

Regularity of the entropy for random walks on hyperbolic groups

François Ledrappier, University of Notre Dame

We consider a hyperbolic group G and random walks defined by probability measures p on G with finite support. We study the regularity of the entropy of the random walk as p varies among probabilities with the same support, and connected problems. We present recent results: the entropy is real analytic if G is a free group, C^1 among symmetric random walks (P. Mathieu 2012) and Lipschitz in general.

Connectedness of Self-affine Sets Associated with 3−digit Sets

King Shun Leung, The Hong Kong Institute of Education

Let $A \in M_2(\mathbb{Z})$ be expanding (all its eigenvalues have modulii greater than 1) with characteritic polynomial $f(x) = x^2 + px \pm 3$. Let $D = \{0, v, kAv + lv\} \subset \mathbb{Z}^2$ be a 3−digit set where $v \in \mathbb{Z}^2 \setminus \{0\}$ and $\{v, Av\}$ is linearly independent. It is well-known that there exists a unique compact set T satisfying

$$T = A^{-1}(T + D) = \left\{ \sum_{i=1}^{\infty} A^{-i} v_i : v_i \in D \right\}.$$

We study the connectedness of T and give a complete characterization of T for the two cases (i) $k = 0$ and (ii) $l = 0$, in terms of l and k respectively.

Diophantine approximation of the orbit of 1 in beta-transformation dynamical system

Bing Li

South China University of Technology and University of Oulu

We consider the distribution of the orbits of the number 1 under the β-transformations T_β as β varies. Mainly, the set of $\beta > 1$ for which a given point can be well approximated by the orbit of 1 is measured by its Hausdorff dimension. More precisely, the dimension of the following set

$$E\big(\{\ell_n\}_{n\geq 1}, x_0\big) = \Big\{ \beta > 1 : |T_\beta^n 1 - x_0| < \beta^{-\ell_n},\ \text{for infinitely many } n \in \mathbb{N} \Big\}$$

is determined, where x_0 is a given point in $[0, 1]$ and $\{\ell_n\}_{n\geq 1}$ is a sequence of integers tending to infinity as $n \to \infty$. For the proof of this result, the notion of the recurrence time of a word in symbolic space is introduced to characterize the lengths and the distribution of cylinders in the parameter space $\{\beta \in \mathbb{R} : \beta > 1\}$.

This is joint work with Baowei Wang.

Hausdorff dimensions of sets related with the Lüroth expansions

Wenxia Li, East China Normal University

We consider the set of real numbers in [0, 1] which have the prescribed group frequencies of digits in their Lüroth expansions. It is proved that the Hausdorff dimension of such a set is equal to the supremum of the Hausdorff dimensions of its subsets with prescribed digit frequencies in their Lüroth expansions.

This is joint work with Yongxin Gui.

An introduction to the theory of minimal sets and their local structure

Xiangyu Liang, Université Paris-Sud 11

A minimal set is a closed set (in an Euclidean space) whose Hausdorff measure cannot be decreased by any local Lipschitz deformation. This notion was invented to give a reasonable model for Plateau's problem, which aims at understanding the behavior of physical objects that admit certain minimizing property, such as soap films. We shall introduce some basic definitions, examples and facts about minimal sets, as well as some recent results and open problems.

Inhomogeneous Diophantine approximation with general error functions

Lingmin Liao, University Paris-East

Let a be an irrational and $\varphi : \mathbb{N} \to \mathbb{R}^+$ be a function decreasing to zero. For any a with a given Diophantine type, we show some sharp estimations for the Hausdorff dimension of the set

$$E_\varphi(a) := \{y \in \mathbb{R} : \|na - y\| < \varphi(n) \text{ for infinitely many } n\},$$

where $\|\cdot\|$ denotes the distance to the nearest integer.

This is a joint work with Michal Rams.

Hole detection and statistical ranking with homologies of digraphs

Yong Lin, People's University of China

We use the theory of homology and cohomology on digraphs introduced recently by A. Grigoryan, Y. Lin, Y. Muranov and S.T. Yau to study the hole detection and statistical ranking problems. These problems have been studied before by using the classic homology and cohomology theory in algebraic topology.

Fractal dimensions of spectrum of Schrödinger operator with Sturm Potential

Qinghui Liu, Beijing Institute of Technology

Let $\alpha \in (0,1)$ be an irrational, and $[0; a_1, a_2, \cdots]$ the continued fraction expansion of α. Let $H_{\alpha,V}$ be the one-dimensional Schrödinger operator with Sturm potential of frequency α, and $\sigma(H_{\alpha,V})$ the spectrum of $H_{\alpha,V}$. We prove, for $V > 26$,

$$\dim_H \ \sigma(H_{\alpha,V}) = s_*, \quad \overline{\dim}_B \ \sigma(H_{\alpha,V}) = s^*,$$

where s_* and s^* are lower and upper pre-dimensions. Under the condition of $\{a_k\}_{k\geq 1}$ be bounded, this formula was proved by Fan, Wen and Liu (Erg. Th. Dyn. Sys.,2011). In this paper we delete the limitation.

Moreover, we show that s_* and s^* are continuous on V for $V > 26$, the limits $s_* \log V$ and $s^* \log V$ exist as V tend to infinity, and the limits are constants depend on $\{a_k\}_{k\geq 1}$.

Mandelbrot's cascade in a random environment

Quansheng Liu, Université de Bretagne - Sud and

Beijing Technology and Business University

Mandelbrot's multiplicative cascade, defined by sums of products of independent random weights, plays an important role in the study of random fractals and related topics. Here we consider the random environment model where the distributions of weights are realizations of a stochastic process (rather then a fixed deterministic distribution), so that the distributions of the weights at n-th level depend on an environment indexed by n. When these random distributions are independent and identically distributed, we show a criterion for the existence of moments of the limit variable Y of Mandelbro's martingale, as well as a criterion for the existence of weighted moments of the form $\mathbb{E}Y^{\alpha}\ell(Y)$, where $\alpha \geq 1$ and ℓ is a positive function slowly varying at ∞. In the deterministic environment, the results improve those of Bingham and Doney (1974, 1975) and Alsmeyer and Rösler (2004) for Galton-Watson processes and Crump-Mode-Jagers processes, of Kahane and Peyrière (1976) and Liu (2000) for Mandelbrot's cascades, and of Alsmeyer and Kuhlbusch (2010) for weighted branching processes. We also show fine results about the local dimension, Hausdorff measure and packing measure of the support of the Mandelbrot's measure defined on the boundary of the associated Galton-Watson tree.

This is joint work with Xingang Liang.

Isodiametric problems with respect to Hausdorff measure

Jun Luo, Zhongshan University

We extend the classical isodiametric problem, with respect to Lebesgue measure, to the cases of Hausdorff measures. Namely, given a closed set $F \subset R^d$ with Hausfdorff dimension s such that restriction m of s-diemsnional Hausdorff measure to F is a Radon measure; we give a criterion that the suprimum of the ratios, $m(X)$ over $|X|^s$, is one when X runs through all subsets of R^d. We call a convex compact set X with $m(X) = |X|^s > 0$ an extremal set; and the isodiametric problem on F is to prove the existence and/or uniqueness of extremal sets and further discuss the structure of such a set, by obtaining information on its shape, its location, and its diameter. If we additionally require that X contain a point $x \in F$, we can infer an isodiametric problem with constraints that is related to the upper convex density of F at x. As examples, we review the isodiametric problem on self-similar sets F with open set condition which have Hausdorff dimension $s \leq 1$. We also mention an interesting case for isodiametric problem with constrains, the case that F is a triangle. Finally, we shall give a short report on recent efforts to study the isodiametric problem on specific sets F with Hausdorff dimension $1 < s < 2$, which largely determine the structure of extremal sets . Such examples are not known from earlier publications of mathematics.

Bi-Affine Fractal Interpolation Functions and their Box Dimension

Peter Massopust,

Technische Universitát München and Helmholtz Zentrum München

We introduce bi-affine interpolation based on a class of bi-affine iterated function systems. We show that the fixed point of the associated Read-Bajraktarević operator defined on an appropriate normed function space is contractive and that its fixed point is a fractal interpolation function whose graph G is the attractor of the bi-affine iterated function system. An explicit formula for the box dimension of G in the case of uniformly spaced interpolation points is presented.

This is joint work with Michael F. Barnsley.

Projection and slicing theorems in Heisenberg groups

Pertti Mattila, University of Helsinki

I shall discuss analogues of Marstrand's projection and line intersection theorems in Heisenberg groups. That is, how do Hausdorff dimension of sets change under natural group projections and what can one say about dimension of intersections with certain families of subgroups?

The talk is based on joint work with Z. Balogh, E. Durand Cartagena, K. Fässler and J. Tyson.

Zeros distribution of derivatives of random polynomials with i.i.d. zeros

Tuen Wai Ng, University of Hong Kong

In this talk, we will consider the problem of finding the zero distributions of the derivatives of random polynomials with i.i.d. zeros following a common distribution supported on a subset of the complex plane.

This is joint work with Pak-Leong Cheung, Jonathan Tsai and Phillip Yam.

Infinite iterated function systems with overlaps

Sze-Man Ngai,

Georgia Southern University and Hunan Normal University

We extend the weak separation condition to infinite iterated function systems of conformal contractions with overlaps. It turns out that there are two natural but different extensions. We study the topological pressure functions associated with these separation conditions. We prove a dimension formula for the Hausdorff dimension of the limit sets under the bounded distortion property and these weak separation conditions.

This is joint work with Ji-Xi Tong.

A multifractal analysis for which Olsen's b and B functions differ

Jacques Peyrière, Université Paris-Sud

One gives an example of a measure for which Olsen's functions b and B differ, and such that the Hausdorff and packing dimensions of the level sets of its local dimension are given by the Legendre transforms of respectively b and B.

Computing Singularity Dimension

Mark Pollicott, University of Warwick

For typical limit sets in the plane generated by a finite set of affine contractions, there is a classical result of Falconer expressing their Hausdorff Dimension in terms of the Singularity Dimension, and another result of Hueter-Lalley describing a smaller open family of contractions for which this equality holds. I will describe an efficient method for estimating the dimension in this latter class.

Sums of random Cantor sets

Michal Rams, Polish Academy of Sciences

We want to present results on geometric properties of random percolations. We will consider percolations with (expected) Hausdorff dimension greater than 1. In this case almost surely all the linear projections of the limit set contain intervals. Even more, almost surely all the projections of the natural measure (except for the projections in the horizontal and vertical directions) is absolutely continuous and with Holder density (this is a joint result with Yuval Peres).

This is the second part of Rams-Simon's joint talk.

Dual Systems of algebraic iterated function systems

Hui Rao, Central China Normal University

We study a class of graph-directed iterated function systems (IFS) on $\mathbb{R}$ with algebraic parameters, which we call algebraic IFS. We construct a dual IFS of an algebraic IFS, and study the duality between the two systems from various points of view. We determine when a dual system satisfies the open set condition, which is fundamental. Under some conditions, a quasi-periodic multiple tiling, which we call the Rauzy-Thurston tiling, of an algebraic IFS is constructed; this tiling extends the previous constructions of Rauzy and Thurston, and it is closely related to the Pisot spectrum conjecture. Finally, we show that the Descarsian product of invariants of the two systems characterizes the points with periodic codings w.r.t. the algebraic IFS; this result has several interesting consequences in number theory.

Either a substitution or a numeration system can define an algebraic IFS in a natural way, and both Rauzy fractals and β-tilings can be obtained as dual IFS. The dual IFS provides a unified and simple framework for the theory of Rauzy fractals, β-tilings and related studies, and allows us gain deeper understanding of the theory.

Zeta Functions and Complex Dimensions of Bounded Sets in Euclidean Space

John A. Rock, Cal Poly Pomona

Motivated by the theory of complex dimensions of (ordinary) fractal strings developed by Lapidus and van Frankenhuijsen in [3], we discuss the development of box-counting zeta functions of [2] and distance zeta functions of [1] (as summarized in [2]) for bounded subsets of Euclidean space. For a given bounded (infinite) set, the abscissae of convergence of these zeta functions coincide with the Minkowski (or box-counting) dimension of the set. Moreover, analysis of the distance zeta function of a bounded set allows for the recovery of some information on the (upper and lower) Minkowski content of the set, and analysis of the box-counting fractal strings of a bounded infinite set allows for a formulation of the underlying box-counting function in terms of its box-counting complex dimensions. These two approaches provide preliminary frameworks for developing theories of complex dimensions for bounded sets.

This is joint work with Michel L. Lapidus and Darko Zubrinic.

Pseudo-differential operators on fractals

Luke Rogers, University of Connecticut

I will talk about joint work with Marius Ionescu and Bob Strichartz in which we study pseudo-differential operators on certain metric-measure spaces that support a Dirichlet form with sub-Gaussian heat-kernel estimates. Elliptic operators are shown to be hypoelliptic in this setting, and the curious quasi-elliptic operators that occur for certain post-critically finite self-similar sets with spectral gaps are found to be elliptic as pseudo-differential operators.

Some progresses on Lipschitz equivalence of self-similar sets

Huojun Ruan, Zhejiang University

In this talk, we will mainly use the algebraic properties of contraction ratios to characterize the Lipschitz equivalence of two dust-like self-similar sets. One interesting result is: we completely characterize the Lipschitz equivalence of dust-like self-similar sets with two branches. If time permits, we will explain our new result on the Lipschitz equivalence of generalized $\{1,3,5\}-\{1,4,5\}$ problem.

This is joint work with Hui Rao, Yang Wang and Li-Feng Xi.

Entropy and geometric measure theory

Tuomas Sahlsten, University of Helsinki

During recent years, the notion of entropy has been fruitful in the study of classical problems in geometric measure theory such as dimensions of projections, and the relationship between dimension and the local distribution of sets and measures. The key tools used here are the *local entropy averages*, which are a convenient way to estimate dimensions. In this talk, we will shortly (try to) demonstrate how local entropy averages can make life easier by using them to sharpen and to unify plenty of classical results related to local distribution such as porosity, conical densities, and local homogeneity.

This is joint work with Pablo Shmerkin (Surrey) and Ville Suomala (Oulu).

Multifractal analysis of some multiple ergodic average – The invariant spectrum

Jörg Schmeling, Lund University

Let (X, T) be a topological dynamical system where T is a continuous map on a compact metric space X. Fürstenberg had initiated the study of the *multiple ergodic average*:

$$\frac{1}{n}\sum_{k=1}^{n} f_1(T^k x) f_2(T^{2k} x) \cdots f_s(T^{sk} x) \tag{0.1}$$

where $f_1, \cdots, f_s$ are s continuous functions on X with $s \geq 2$ when he proved the existence of arithmetic sequences of arbitrary length amongst sets of integers with positive density. Later on, the research of such a kind of average has attributed a lot of attentions.

We study the multiple ergodic averages

$$\frac{1}{n}\sum_{k=1}^{n} \varphi(x_k, x_{kq}, \cdots, x_{kq^{\ell-1}})$$

on the symbolic space $\Sigma_m = \{0, 1, \cdots, m-1\}^{\mathbb{N}^*}$ where $m \geq 2, \ell \geq 2, q \geq 2$ are integers. In his talk Ai-Hua Fan gives a complete solution to the problem of multifractal analysis of the limit of the multiple ergodic averages.

We will continue by considering the invariant part of the multifractal level sets, i.e. we will study the maximal dimension of an invariant or multiple mixing measure supported on these level sets. Here many new interesting phenomena occur. In general there will be no invariant measure with the same dimension as the level sets. Moreover the invariant and the mixing spectra differ. On the other hand we will point on some connections to probability theory (von Mise statistics), ergodic optimization of multiple integrals and also indicate some new phase transition phenomena.

This is joint work with Ai-Hua Fan, and Meng Wu.

Hardy-Littlewood series and (even) continued fractions

Stéphane Seuret, Université Paris-Est

In this talk, we review some old results and give some new theorems on the local regularity analysis of Hardy-Littlewood series

$$R_\alpha(x) = \sum_{n\geq 1} \frac{\sin(\pi n^2 x)}{n^\alpha}.$$

When $\alpha = 2$, one recovers the Riemann "non-differentiable" function, whose differentiability and multifractal properties have been widely studied for a hundred years (by Hardy, Littlewood, Gerver and Jaffard).

When $1 < \alpha < 2$, R_α has the same properties as R_2.

When $\alpha > 1$ and $\sin(2\pi n^2)$ is replaced by $\sin(2\pi P(n))$ for some polynomial with degree ≥ 2, Chamizo and Ubis obtained recently lower and upper bounds for the multifractal spectrum of the corresponding function.

When $1/2 < \alpha \leq 1$, R_α belongs to L^2, hence converges almost everywhere by Carleson's theorem, but not everywhere. We give some Diophantine conditions on x that guarantees the convergence of R_α. These Diophantine conditions are expressed in terms of the even continued fraction expansion of a real number x, which is associated with the dynamical system $x \mapsto -\frac{1}{x} \mod 2$ (instead of the Gauss map $x \mapsto \frac{1}{x} \mod 1$

for the standard continued fraction expansion). To get the Diophantine conditions, convergence results for the even continued fractions expansion by Kraikamp, Lopes, Sinai, are extended.

Fractional Lévy Processes: Paths, Dimensions, and Related

Narn-Rueih Shieh, National Taiwan University

In this talk, we report some progress for paths and dimensions of the harmonizable and the linear fractional processes driven by a non-Gaussian Lévy process with, say, exponential moment. We show that these two processes are different in law, due to different modulus of continuity. We present a dimension formula of $X(E)$ in the harmonizable case. We propose the multi-fractional and the exponential processes associated with such fractional processes. We also propose the relativistic stable motion, that is, the subordination of BM by a relativistic stable-subordinator, as a good candidate for such driving LP, since it meets the large and the small scaling features of fLP, which are lack for fBM. This talk is adapted from joint works with several co-authors; confer to URL: `http://www.math.ntu.edu.tw/~shiehnr` .

Continuity of subadditive pressure

Pablo Shmerkin, University of Surrey

The Hausdorff dimension of sets invariant under conformal dynamical systems can often be realized as the zero of certain natural pressure equation (going back to Bowen). This pressure is usually continuous as a function of the defining dynamics, in the appropriate topology, and hence so is the Hausdorff dimension of the invariant set.

The situation is dramatically more complicated in the non-conformal situation where, nevertheless, a subadditive version of the pressure equation, involving singular values of a matrix cocycle, is crucial. A natural question is therefore whether this subadditive pressure is also a continuous function of the dynamics (or, what is the same, of the associated cocycle). We resolve this in the affirmative in many important situations, in particular answering a question of Falconer and Sloan.

This is joint work with De-Jun Feng.

Projections of Mandelbrot percolations

Károly Simon, Technical University of Budapest

We want to present results on geometric properties of random percolations. We will present results for percolations with (expected) Hausdorff dimension smaller than 1. In this case almost surely all the linear projections of the limit set have the same Hausdorff dimensions as the limit set itself. The method used in this proof can be surprisingly applied to the problem of sums of n independent random Cantor sets (the case for $n = 2$ was done by Dekking and Simon). We prove that if the sum of their expected Hausdorff dimensions is greater than 1 then almost surely their algebraic sum will contain an interval. Another application is for distance sets: for a percolation with expected Hausdorff dimension greater than 1/2 its distance set almost surely contains an interval.

This is the first part of Rams-Simon's joint talk.

Iterated function systems with a given continuous stationary distribution

Örjan Stenflo, Uppsala University

Let μ be an arbitrary continuous probability distribution on $\mathbb{R}$. In this talk we show how to construct an IFS with probabilities having μ as its unique invariant probability measure.

New developments of fractal PDE

Weiyi Su, Nanjing University

Fractal PDE, as a quite new topic in the area of Fractal Analysis, is developing very fast since the end of last century. In this talk, we will show 4 important ideas to study fractal PDE, and the main methods, main results of them. Some open problems are also indicated.

The abc-problem for Gabor system

Qiyu Sun, University of Central Florida

One of fundamental problems in Gabor theory is to identify window functions ϕ and time-frequency shift sets $a\mathbf{Z} \times \mathbf{Z}/b$ such that the corresponding Gabor system

$$\mathcal{G}(\phi, a\mathbf{Z} \times \mathbf{Z}/b) := \{e^{-2\pi int/b}\phi(t - ma) : \ (m, n) \in \mathbf{Z} \times \mathbf{Z}\}$$

is a Gabor frame for $L^2(\mathbf{R})$, the space of all square-integrable functions on the real line $\mathbf{R}$. The range of density parameters a and b such that the Gabor system $\mathcal{G}(\phi, a\mathbf{Z} \times \mathbf{Z}/b)$ is a frame for $L^2(\mathbf{R})$ is fully known surprisingly only for few window functions ϕ and could be arbitrarily complicated. In this talk, we will introduce maximal invariant sets of some piecewise linear transformations, study the dynamic system associated with the piecewise linear transformations, explore topological, algebraic and covering properties of maximal invariant sets, and finally give the full classification of triple (a, b, c) of positive numbers (i.e., the *abc*-problem for Gabor systems) such that the Gabor system $\mathcal{G}(\chi_I, a\mathbf{Z} \times \mathbf{Z}/b)$ generated by the ideal window function χ_I on an interval of length c is a Gabor frame for $L^2(\mathbf{R})$.

Intersection properties of random and deterministic measures

Ville Suomala, University of Oulu

Projection properties of various random fractals have gained a lot of interest in the last few years. In this talk, I will discuss some random measures and their intersection properties with deterministic measures. Results on orthogonal projections are obtained as corollaries.

Let $\nu_{tt\in\Gamma}$ be a parametrized family of measures (e.g. the length measures on all lines of the plane). It turns out that for certain random martingale measures μ, it is a.s. possible to define the intersection of μ and ν_t (denoted by Y^t) for all $t \in \Gamma$. In our main results, we give sufficient conditions for the a.s. continuity of $t \mapsto Y^t$. As a corollary, we find several families of random fractal measures, for which there are no exceptional directions in the Marstrand projection theorem. Furthermore, for any $k, n \in \mathbb{N}$ and $k < s < n$, we verify the existence of measures μ of dimension s such that all orthogonal projections of μ onto k-dimensional linear subspaces of $\mathbb{R}^n$ are (jointly) Hölder continuous.

Two main classes of random measures satisfying our general assumptions are certain Poissonian cut-out measures and limit measures on random subdivision fractals (generalizing fractal percolation measures).

This is joint work with Pablo Shmerkin.

Fractal tiles and quasidisks

Tai-Man Tang, Xiangtan University

We investigate planar self-affine tiles and self-similar tiles. We show that a self-affine tile may not be the closure of a quasi-disk. For self-similar tiles, we show that if the distances between the vertices in the tiling generated by the tile in a certain manner has a positive lower bound, the tile is the closure of a quasidisk. A quasidisk has many characterizing properties, some of which have simple descriptions. By showing that the tiles are quasidisks, we prove that the tiles have these properties.

This is joint work with Sze-Man Ngai.

Diffusive limits on the Penrose tiling

András Telcs, University of Pannonia

In this lecture random walks on the Penrose tiling are investigated. Heat kernel estimates and the invariance principle are shown proving Domokos Szasz's conjecture.

Spectral and vector analysis on fractafolds

Alexander Teplyaev, University of Connecticut

A fractafold, a space that is locally modeled on a fractal, is the fractal equivalent of a manifold. This notion was introduced by Strichartz, who showed how to compute the discrete spectrum of the Laplacian on compact Sierpinski fractafolds in terms of the spectrum of finite graph Laplacians, in particular producing isospectral fractafolds. In a joint work with Strichartz it was furthermore shown how to extend these results for unbounded fractafolds which have continuous spectrum. In parallel to the spectral analysis, a recent progress was made in understanding differential forms and vector analysis on fractafolds and more general spaces. For instance, self-adjointness of the magnetic Laplacian, the Hodge theorem, and the existence and uniqueness for the Navier-Stokes equations have been proved (jointly with Michael Hinz) for topologically one-dimensional spaces with strong local Dirichlet forms that can have arbitrary large Hausdorff and spectral dimensions. These and related joint results with Marius Ionescu, Dan Kelleher, Luke Rogers, Michael Roeckner will be discussed.

Gaussian free fields on self-similar fractals

Baris Ugurcan, Cornell University

In the last few decades, there has been tremendous amount of work which focuses on generalizing the analysis and stochastic processes to fractals. Equipped with this machinery, we undertake a systematic investigation of Gaussian Free Fields (GFFs) on fractals including the Sierpinski Gasket and generalized Sierpinski Carpets. The covariance structure of the GFF thereon, is the Green's function of the Laplacian on the fractal. By using the standard results on heat kernel estimates and spectral asymptotics, we identify the associated Cameron-Martin space. Similar to the dichotomy in the Euclidean case, we show that the GFFs on fractals with $d_s < 2$ are continuous random functions whereas GFF should be realized as a random distribution when $d_s \geq 2$. In this talk, I will go through these results, then explain other fine properties we obtained for the GFFs on fractals, including regularity and expected maxima. I will point out how the various quantities related to GFFs scale on fractals and how Hausdorff dimension replaces Euclidean dimension while talking about regularity properties.

This is joint work with Joe P. Chen (Cornell).

Localized Birkhoff average in beta dynamical systems

Baowei Wang, Huazhong University of Science and Technology

In this talk, we will discuss the localized multifractal spectrum of Birkhoff average in the beta-dynamical system $([0,1], T_\beta)$ for general $\beta > 1$, namely the Hausdorff dimension of the following level sets

$$\Big\{x \in [0,1] : \lim_{n\to\infty} \frac{1}{n}\sum_{j=0}^{n-1} \psi(T^j x) = f(x)\Big\},$$

where f and ψ are two continuous functions defined on the unit interval $[0,1]$. Instead of a constant function in the classical multifractal cases, the function f here varies with x. The method adopted in the proof indicates that as far as the multifractal analysis of Birkhoff average is concerned, the multifracal spectrum can be achieved via approximating the system by its subsystems which are of finite type.

Boundary theory and self-similar set

Xiang-Yang Wang, Sun Yat-Sen University

We study the induced graphs by iterated function systems (IFS). We prove that if the IFS satisfying the weak separation condition or the self-similar sets have positive Lebsgue measures then the induced graphs are hyperbolic in the sense of Gromov. We also study the the random walks on the above induced graphs, under some mild conditions, we prove that the Martin boundaries, the hyperbolic boundaries and the self-similar sets are homeomorphic.

Open set condition for graph directed self-similar structure

Zhiying Wen, Tsinghua University

Graph directed self-similar structure generalizes the concept of self-similar set and contains some important fractal sets for some general metric space. We characterize the open set condition, which is fundamental in the study of self-similar set, for graph directed self- similar structure in terms of the post critical set. Using this characterization, we establish the relations between open set condition and other separation conditions including postcritically finite, finitely ramified condition and finite preimage property. It turns out that whether the intrinsic metric is doubling makes difference. In particular, finitely ramified condition implies open set condition in case of doubling metric but does not in case of non-doubling metric.

Packing Dimension Results for Anisotropic Gaussian Random Fields

Dongsheng Wu, University of Alabama in Huntsville

Let $X = \{X(t), t \in \mathbf{R}^N\}$ be a Gaussian random field with values in $\mathbf{R}^d$ defined by

$$X(t) = \big(X_1(t), \ldots, X_d(t)\big), \qquad \forall\, t \in \mathbf{R}^N,$$

where $X_1, \ldots, X_d$ are independent copies of a centered real-valued anisotropic Gaussian random field X_0. In this talk, we present some packing dimension results for the range $X(E)$, where $E \subseteq \mathbf{R}^N$ is a Borel set. For this purpose we extend the original notion of packing dimension profile due to Falconer and Howroyd (1997) to the anisotropic metric space $(\mathbf{R}^N, \rho)$, where $\rho(s,t) = \sum_{j=1}^N |s_j - t_j|^{H_j}$ and $(H_1, \ldots, H_N) \in (0,1)^N$ is a given vector. We believe that the extended notion of packing dimension profile is of independent interest.

This talk is based on a joint work with Anne Estrade and Yimin Xiao.

The dimensional theory of continued fractions

Jun Wu, Huazhong University of Science and Technology

Continued fraction expansion lies in the study of expansions of real numbers. It appears in many different fields of mathematics: Diophantine approximation, probability theory, dynamical systems etc. Measure-theoretic and dynamical properties of continued fraction and their applications to Diophantine approximation have been extensively studied. In this talk, we collected some dimensional results occurring in continued fraction expansion.

Brownian Motion and Thermal Capacity

Yimin Xiao, Michigan State University

Let $W = \{W(t), t \in \mathbb{R}_+\}$ denote d-dimensional Brownian motion. We find an explicit formula for the essential supremum of Hausdorff dimension of $W(E) \cap F$, where $E \subset (0\,, \infty)$ and $F \subset \mathbb{R}^d$ are arbitrary nonrandom compact sets. Our formula is related intimately to the thermal capacity of Watson (1978). We prove also that when $d \geq 2$, our formula can be described in terms of the Hausdorff dimension of $E \times F$, where $E \times F$ is viewed as a subspace of space time.

This is joint work with Davar Khoshnevisan.

Ideal class and Lipschitz equivalent class

Ying Xiong, South China University of Technology

This paper concerns the Lipschitz equivalence of totally disconnected self-similar sets in $\mathbb{R}^d$ satisfying the OSC and with commensurable ratios. We obtain the complete Lipschitz invariants for such self-similar sets. The key invariant we found is an ideal related to the IFS. This discovery establishes a bijection between Lipschitz equivalence of similar set and ideal class in algebraic number theory.

Consequently, we can prove a finiteness result about the Lipschitz equivalent class. Furthermore, if the related ring is a principle ideal domain, then there is only one Lipschitz equivalent class, all sets in this class are Lipschitz equivalent to a symbolic metric space.

A generalization of Jarnik-Besicovitch Theorem by continued fraction

Jian Xu, Huazhong University of Science and Technology

The classical Jarnik-Besicovitch sets, expressed in terms of continued fractions, can be written as

$$\left\{x \in [0,1] \colon a_{n+1}(x) \geq e^{-\tau(\log|T'x| + \cdots + \log|T'(T^{n-1}x)|)},\ \text{i.o. } n \in \mathbb{N}\right\},$$

where T is the Gauss map and $a_n(x)$ are the partial quotients of x. We consider the size of the following generalized Jarnik-Besicovitch set

$$\left\{x \in [0,1] \colon a_{n+1}(x) \geq e^{f(x)(g(x) + \cdots + g(T^{n-1}x))},\ \text{i.o. } n \in \mathbb{N}\right\},$$

where $f, g \colon [0,1] \to \mathbb{R}^+$ are positive functions.

Conformal Invariance of the Exploration Path in 2D Critical Bond Percolation in the Square Lattice

S.C.P. Yam, The Chinese University of Hong Kong

In this talk, I shall outline a proof of the convergence of the critical bond percolation exploration process on the square lattice to the trace of SLE6. This is an important conjecture in mathematical physics and probability. The case of critical site percolation on the hexagonal lattice was established in the seminal work of Smirnov via proving Cardys formula. However, our proof relies on a series of transformations that allow us to apply the convergence in the site percolation case on the hexagonal lattice to obtain certain estimates that is enough for us to prove the convergence in the case of bond percolation on the square lattice.

Three-dimensional flows: several phenomena

Dawei Yang, Jilin University

I would like to talk about something about several typical phenomena for three-dimensional singular flows. The typical phenomena include Morse-Smale systems, systems with a transverse homoclinic intersection of some hyperbolic periodic orbit, homoclinic tangencies, Lorenz-like systems. I will also recall the results for non-singular flows.

Veech problem and its higher order form

Xiangdong Ye, University of Science and Technology of China

An old problem in combinatorial number theory which is to know whether any difference set $S - S$, where S is a subset of $\mathbb{Z}$ with bounded gaps, must contain a Bohr neighborhood of zero. It is known by a result of Veech that this is true up to a set of density zero, but it is not known whether this set can be dispensed with. We will present results related to its higher order form, which is a joint work with W. Huang and S. Shao.

Ruelle Operator with weakly contractive IFS

Yuanling Ye, South China Normal University

The Ruelle operator has been studied extensively both in dynamical systems and iterated function systems (IFS). Given a weakly contractive IFS $(X, \{w_j\}_{j=1}^m)$ and an associated family of positive continuous potential functions $\{p_j\}_{j=1}^m$, a triple system $(X, \{w_j\}_{j=1}^m, \{p_j\}_{j=1}^m)$ is set up. In this paper we study Ruelle operators associated to the triple systems. The paper presents an easily verified condition. Under this condition, the Ruelle operator theorem holds provided that the potential functions are Dini continuous. Under the same condition, the Ruelle operator is quasi-compact, and the iterations sequence of the Ruelle operator converges with a specific geometric rate, if the potential functions are Lipschitz continuous.

Large Some applications of fractal methods to geomagnetic data analysis

Zuguo Yu, Xiangtan University & Queensland University of Technology

In this talk, I would like to introduce some applications of fractal methods to geomagnetic data analysis done by my collaborators and me in the past a few years.

First, we proposed measure representations of D_{st} data and used the recurrent function systems (RIFS) model to simulate them. The RIFS simulation leads to a method to predict storm patterns included in its attractor.

Then we proposed a two-dimensional chaos game representation (CGR) for the D_{st} index. The CGR provides an effective method to characterize the multifractality of the D_{st} time series. The probability measure of this representation was then modelled as a 2-dimensional RIFS.

Third, in a continuous-time framework, it is known that long range dependence (LRD) can be obtained by replacing ordinary differential operators by fractional differential operators in differential equations driven by white noise. We introduced the application of a class of general fractional differential equations (FDE) driven by Levy noise to magnetic AE and B_x data analysis. We found that inverse Gaussian distribution can fit the Levy noise term for the AE data well and alpha-stable distribution can fit the Levy noise term for the B_x data. Then a method was introduced to estimate the parameters in the FDE. Then the solution of the FDE can be used to simulate the AE data time series. With the help of empirical mode decomposition (EMD), the solution of the FDE can be used to simulate the B_x data time series.

Forth, we did multifractal analysis of D_{st}, a_p and solar brightness X_l data and discussed class dependence of geomagnetic storm and solar flare. The multifractal properties of the daily solar X-ray brightness, X_l and X_s, during the period from 1 January 1986 to 31 December 2007 which includes two solar cycles were examined using the universal multifractal approach and multifractal detrended fluctuation analysis (MF-DFA). Then we converted these time series into networks using the horizontal visibility graph technique. It is found that the empirical $K(q)$ curves of raw time series can be fitted by the universal mulitifractal model. The multifractal scaling for the networks of the time series can reflect some properties which cannot be picked up by using the same analysis on the original time series. This suggests a potentially useful method to explore geophysical data.

Last, we have examined the anomalies in zonal averages of total ozone and of temperatures at the Earth's surface during 32 years of observations using their visibility networks and found that the latitude dependence of both measurements are similar enough to suggest that they are related.

Approximation of fractals by tubular neighborhoods - geometric and analytic properties

Martina Zähle, Friedrich Schiller University Jena

Fractal curvatures arise as rescaled (average) limits of classical versions for parallel sets of small distances. We give a survey on classes of fractals, for which this has been proved. Moreover, the geometry of the approximating sets is reflected in the analytic behaviour of associated spectral structures. For certain types of self-similar sets asymptotic properties of related Dirichlet forms are considered.

Slices through self-similar sets

Rüdiger Zeller, Arndt University of Greifswald

Intersections of finite type of Sierpinski carpet and lines were studied by Li (1997) and Xi and Wen (2010) and ergodic theorems for slices through Sierpinski gasket and Sierpinski carpet were recently proved by Simon, Manning, Bárány and Ferguson. We consider finite type intersections of hyperplanes and self-similar sets in $\mathbb{R}^n$, where the IFS is given by functions of the form $f_j(x) = \frac{1}{\beta^{d_j}}(x + v_j)$, so that $d_j \in \mathbb{N}$, $v_j \in \mathbb{R}^n$, and β is a Pisot number. A characterisation of finite type intersections is given and a number of three-dimensional examples are studied in order to determine their geometry. From the algebraic point of view our result generalises a theorem on β-representations proved by Schmidt in 1979.

Modeling potential as fiber entropy and pressure as entropy

Guohua Zhang, Fudan University

We try to throw some new light on topological pressure by proving that, given a topological dynamical system, every nonnegative, upper semicontinuous and subadditive potential is nearly equal to the fiber entropy potential in some relatively symbolic extension of the system. Based on this, for such a potential we could provide a new proof of the Variational Principle for Pressure by reducing it to the usual Variational Principle (for entropy) applied to the extended system. We also provide examples showing that both assumptions, continuity and additivity, under which defining topological pressure equivalently using spanning sets, are essential.

This is joint work with T. Downarowicz and D. Huczek.

Discrete Fractal Dimensions of the Ranges of Random Walks in $\mathbb{Z}^d$ Associate with Random Conductances

Xinghua Zheng, Hong Kong University of Science and Technology

Let $X = \{X_t, t \geq 0\}$ be a continuous time random walk in an environment of i.i.d. random conductances $\{\mu_e \in [1, \infty), e \in E_d\}$, where E_d is the set of nonoriented nearest neighbor bonds on the Euclidean lattice $\mathbb{Z}^d$ and $d \geq 3$. Let $\mathrm{R} = \{x \in \mathbb{Z}^d : X_t = x \text{ for some } t \geq 0\}$ be the range of X. It is proved that, for almost every realization of the environment, $\dim_{\mathrm{H}} \mathrm{R} = \dim_{\mathrm{P}} \mathrm{R} = 2$ almost surely, where $\dim_{\mathrm{H}}$ and $\dim_{\mathrm{P}}$ denote respectively the discrete Hausdorff and packing dimension. Furthermore, given any set $A \subseteq \mathbb{Z}^d$, a criterion for A to be hit by X_t for arbitrarily large $t > 0$ is given in terms of $\dim_{\mathrm{H}} A$. Similar results for Bouchoud's trap model in $\mathbb{Z}^d$ $(d \geq 3)$ are also proven.

This is based on joint work with Yimin Xiao.