

MATH4060 HW3 Due date: Mar 2, 2021, 12:00pm

1. Show that

(a) $\int_1^{\infty} e^{-t} t^{s-1} dt$ defines an entire function.

(b) $\forall \varepsilon > 0, \exists C > 0$ such that

$$|s| |\log |s|| \leq C |s|^{1+\varepsilon} \quad \forall s \in \mathbb{C} \setminus \{0\}.$$

2. (a) Exercise 13 of Chapter 6 of the Textbook.

(b) Using part (a), show that

$$\Gamma(s) \Gamma(s + \frac{1}{2}) = \sqrt{\pi} 2^{1-2s} \Gamma(2s) \quad \text{--- (*)}$$

3. Exercise 11 of Chapter 6 of the Textbook.

(Misprints in the Textbook: (i) "strip" should be $\{x+iy = |y| < \frac{\pi}{2}\}$
(ii) $\hat{f}(\xi) = \Gamma(a - 2\pi i \xi)$)

4. Exercise 12 of Chapter 6 of the Textbook. (Use formula (*) above)

5. Exercise 15 of Chapter 6 of the Textbook.

(End)