MATH3060 HW8 Due date: No need to hand-in

1. Use Baire Category Theorem to show that transcendental numbers are dense in $\mathbb{R}$. (Recall that a number is called algebraic if it is a root of some polynomial with integer coefficients, and a number is called transcendental if it is not algebraic.)

2. Show that any norm on $\mathbb{R}^n$ is equivalent to the usual Euclidean norm $\|\cdot\|_2$ defined by $\|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$ for $x = (x_1, \cdots, x_n) \in \mathbb{R}^n$.

3. Let X be a metric space, G_n, $n = 1, 2, \cdots$, are open subsets. Suppose that $G = \bigcap_{n=1}^{\infty} G_n$ is dense in X. Show that G is a residual.

4. Let $\mathcal{C}$ be the set of convergent sequences with d_∞ metric, and $\mathcal{C}_{\mathbb{Q}} = \{ x = \{x_n\} \in \mathcal{C} : \lim_{n\to\infty} x_n \in \mathbb{Q} \}$.

   Is $\mathcal{C}_{\mathbb{Q}}$ nowhere dense in $\mathcal{C}$? Is $\mathcal{C}_{\mathbb{Q}}$ of 1st category in $\mathcal{C}$?

(End)