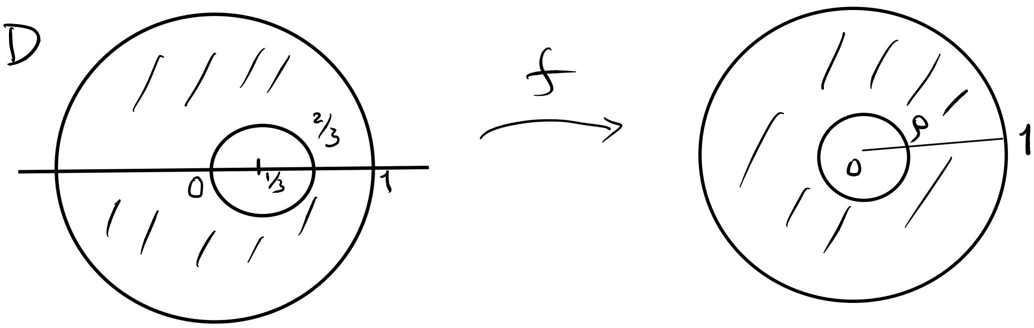
HW 9 Due Apr 20, 2017

1. (a) Find a linear fractional transformation $w = f(z)$ such that f maps domain D (figure on the left)



to an annulus of the form $\rho < |z| < 1$ for some $0 < \rho < 1$. (Note $z = \frac{1}{3} \not\mapsto w = 0$)

(b) Then find a harmonic function u defined on D such that $u \equiv 0$ on the inner circle and $u \equiv 1$ on the outer circle. (Hint: Use (a) & $\log|w|$)

2. (a) Show that $h(u,v) = e^{-v}\sin u$ is harmonic for all $(u,v) \in \mathbb{R}^2$.

(b) Using the transformation $w = z^2$, show that $H(x,y) = e^{-2xy}\sin(x^2 - y^2)$ is harmonic in the quadrant $\{x+iy : x>0, y>0\}$

3. Show that g is a conformal self-map of the upper half-plane $\{x+iy : y>0\}$ if and only if

$$g(z) = \frac{az+b}{cz+d} \quad \text{with } a,b,c,d \in \mathbb{R} \text{ \& } ad-bc>0.$$

Moreover, show that these $g(z)$ can be normalized to have $ad-bc=1$.

4. Let f be an analytic function defined on $\{|z|<1\}$ with $\operatorname{Im} f(z) > 0$. Show that

$$|f'(0)| \leq 2 \operatorname{Im} f(0).$$