MATH 2550

Home Work 1

(Additional Questions)

(Due date: To be announced)

The following exercises aim to review stuff you have learned so far.

1. (About planes in the 3D space) Consider the plane containing the 3 position vectors $(1,0,-1)^t, (0,0,1)^t$ and $(0,1,0)^t$. Find an equation of this plane in the form $Ax + By + Cz = 1$ (if any).
2. (About planes in the 3D space) The equation $x - 3y + 4z = 2$ is the equation of a plane in the 3D space. Answer the following questions (i) Is the (position) vector $\vec{r}_0$ given by the formula $\vec{r}_0 = 1\hat{\imath} + 1\hat{\jmath} + 1\hat{k}$ the position vector of a point on this plane? (Just answer Yes or No & Explain why you chose "Yes" or "no") (ii) Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ be the position vector of any other point on this plane. Rewrite the equation of this plane in the form
$$\{\vec{r} - (1\hat{\imath} + 1\hat{\jmath} + 1\hat{k})\} \cdot \vec{N} = 0$$
by finding some normal vector $\vec{N}$.
3. (About partial derivative) Let $f(x, y) = \sin(xy^2 + x^2y)$ be a function of two variables x and y. Compute (i) $\frac{\partial f}{\partial x}$ (another notation for this "partial derivative of f with respect to the variable x is f_x). (ii) Compute $\frac{\partial f}{\partial y}$.
4. (Geometric Meaning of a certain vector) Let $z = x^2 + 2y^2$ be a surface in the 3D space and we rewrite this equation in the form $x^2 + 2y^2 - z = 0$. What is the "geometric" meaning of the vector $\frac{\partial x^2}{\partial x}\vec{\imath} + \frac{\partial(2y^2)}{\partial y}\vec{\jmath} + \frac{\partial(-z)}{\partial z}\vec{k}$? (You only need to write a sentence about the "geometric meaning" of this vector!)
5. (About tangent line function of one variable) In school calculus, we learned about "tangent line" to a function $f(x)$. The tangent line at $x = c$ satisfies this equation: $y = f(c) + f'(c) \times (x - c)$. Answer the following questions: (i) If $f(x) = \frac{1}{x}$ and $c = 1/2$, what is the equation of the tangent line at the point $x = 1/2$? (ii) Find the x −intercept of this tangent line with the positive x −axis, (iii) Find the y −intercept of this tangent line with the positive y −axis.

6. (About tangent plane and function of two variables) In our course, we mentioned that if we have a function of two variables x and y, the equation of the tangent plane at the point $x = a, y = b$ is given by the equation $z = f(a, b) + f_x(a, b) \times (x - a) + f_y(a, b) \times (y - b)$. Answer the following questions: (i) If we have the function $f(x, y) = \frac{1}{xy}$, and consider the point $x = 1, y = 2$, what is the equation of the tangent plane at this point? (ii) sketch this tangent plane.

7. (In this question, you will use Green's Theorem to calculate the area of a region in the xy −plane). Green's Theorem says: For a "nice" region R, the following formula holds: $\int_C Adx + Bdy = \iint_R (B_x - A_y)dxdy$. (Here both A and B are functions of x and y). Answer now the following questions: (i) If we now consider the functions $A(x, y) = -y, B(x, y) = x$, what will the above formula for Green's Theorem produce?, (ii) Use what you have just obtained and assuming that now

$$R = \text{circlar disk of radius 3 with center at } (0,0),$$

what is now the curve C ? (iii) find the value of $\int_C -ydx + xdx$, (iv) compute $\int_C -ydx + xdx$ directly (without using Green's Theorem) by "parametrizing" the curve C.