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CW2.

1(a) $\vec{r}$ denotes (any) position vector on the plane given by

$$(\vec{r}-\vec{r}_0)\cdot\vec{N}=0 \qquad (*)$$

(b) $\vec{r}_0$ denotes (one) position vector on the plane $(*)$.

(c) $\vec{N}$ is (not) unique.

If $\vec{N}$ is a normal vector, then $k\vec{N}$ is also a normal vector, where $k$ is (any) (non-zero) real no.

(d) Let $\vec{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$. We know that $\vec{N}=\begin{pmatrix}1\\1\\1\end{pmatrix}$ & $\vec{r}_0=\begin{pmatrix}0\\0\\0\end{pmatrix}$ $\boxed{\because \text{the plane passes through } \begin{pmatrix}0\\0\\0\end{pmatrix}.}$ Therefore

$(*)$ takes the form

$$\left(\underbrace{\begin{pmatrix}x\\y\\z\end{pmatrix}}_{\vec{r}}-\underbrace{\begin{pmatrix}0\\0\\0\end{pmatrix}}_{\vec{r}_0}\right)\underbrace{\cdot}_{\text{dot product}}\underbrace{\begin{pmatrix}1\\1\\1\end{pmatrix}}_{\vec{N}}=0 \qquad (1)$$

After simplification, we get

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=0$$

i.e. $x+y+z=0$.

2. (a) The plane $x-2y+3z=1$ does (not) pass through the origin, because

$x=0,\ y=0,\ z=0$

doesn't satisfy the equation, since

$$\underbrace{0-2.0+3.0}_{=0} \neq 1$$

(b) Rewriting the equation of the plane in the form

$$\underbrace{x-2y+3z-1}_{F(x,y,z)} = 0$$

Name the left-hand side as $F(x,y,z)$.

Then we learned from one of the lectures that

$\vec{\nabla}F$ is a ~~the~~ normal vector to this plane.

But what is $\vec{\nabla}F$ ?

$$\vec{\nabla}F = \begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \\ \frac{\partial F}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{\partial(x-2y+3z-1)}{\partial x} \\ \frac{\partial(x-2y+3z-1)}{\partial y} \\ \frac{\partial(x-2y+3z-1)}{\partial z} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

Therefore ANS: $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

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(c) Since $\vec{N} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

If we (denote) (any) (position) vector $\vec{r}$ on the plane by $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, then

"all we need to do is to find (one) (position) vector on the plane"!

This is done by trial-and-error!

Since $x-2y+3z=1$

If we put $x=0$, $y=0$, then $z=\frac{1}{3}$. Therefore we can choose $\vec{r}_0 = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{3} \end{pmatrix}$.

Then we have

$$\left(\underset{\vec{r}}{\begin{pmatrix}x\\y\\z\end{pmatrix}}-\underset{\vec{r}_0}{\begin{pmatrix}0\\0\\\frac{1}{3}\end{pmatrix}}\right)\cdot\underset{\vec{N}}{\begin{pmatrix}1\\-2\\3\end{pmatrix}}=0$$

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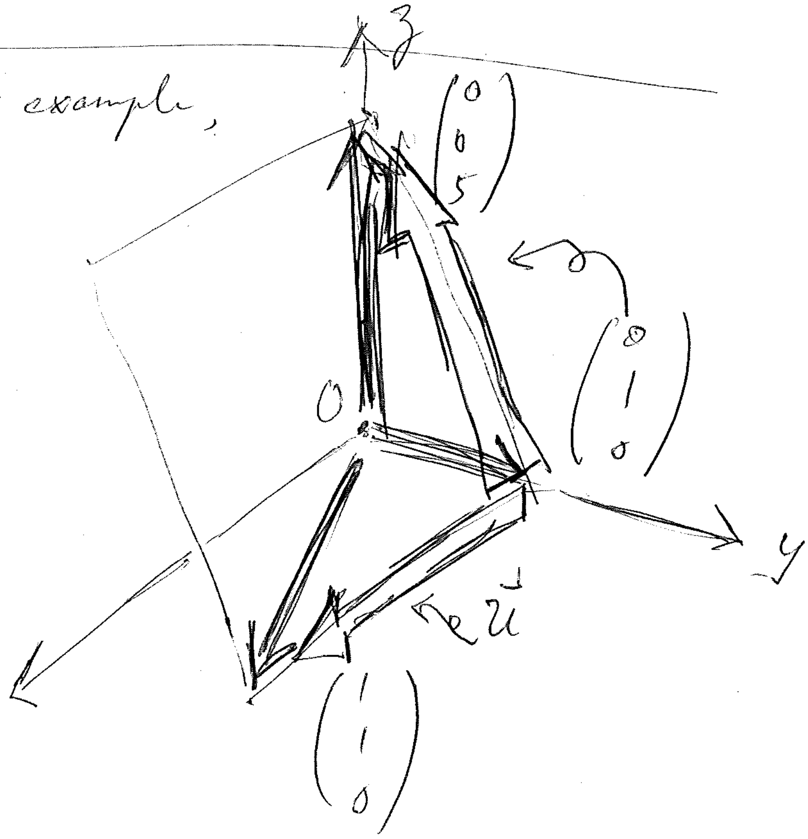
3. We can choose, for example, the vector $\vec{u}$ & $\vec{v}$.




$$\vec{u}=\begin{pmatrix}1\\1\\0\end{pmatrix}-\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$=\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\vec{v}=\begin{pmatrix}0\\0\\5\end{pmatrix}-\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\-1\\5\end{pmatrix}.$$