THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1010H University Mathematics 2014-2015
Test 2, 19 Mar, 2015

- Time allowed: 45 minutes
- Answer all questions.
- Show your work clearly and concisely in your answer book.
- Write down your name and student ID number on the front page of your answer book.
- You are allowed to use a calculator in this test.

1. By using L'Hôpital Rule, find

   (a) $\lim_{x\to 1/2} \dfrac{\cos^2 \pi x}{e^{2x} - 2ex}$

   (b) $\lim_{x\to 0} (\sin x)^{\tan x}$

(20 points)

2. By using implicit differentiation, find $\dfrac{d}{dx} tan^{-1} x$.

(15 points)

3. Write down the Taylor polynomial $P_3(x)$ of degree 3 generated by $f(x) = \ln(1+x)$ at 0, and hence approximate $\ln 1.01$.

(15 points)

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4. (a) Let $0 < k < 1$. Show that

$$(1-k)t + k \geq t^{1-k} \text{ for all } t > 0.$$

(b) Hence, deduce that

$$(1-k)r + ks \geq r^{1-k}s^k \text{ for all } r,\ s > 0.$$

(20 points)

5. Let $f(x) = xe^{-x^2}$, where x is a real number.

(a) Find $f'(x)$ and $f''(x)$

(b) Find the range of x such that

(i) $f'(x) > 0$

(ii) $f'(x) < 0$

(iii) $f''(x) > 0$

(iv) $f''(x) < 0$

(c) Find the local extrema and saddle points, if any.

(d) Find the points of inflection, if any.

(e) Find the asymptotes of the graph of $f(x)$, if any.

(f) Sketch the graph of $f(x)$.

(30 points)