Math4230 Exercise 9

1. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function. Suppose x^* is a local minimizer of f, show that it is also a global minimizer.

2. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a strictly convex function. Suppose f has a global minimizer, show that it is unique.

3. Let $f : Y \to \mathbb{R}$ be a Lipschitz continuous function with constant L. Let X be a nonempty closed subset of Y, and c be a number such that $c > L$.
   (a) Show that if x^* minimizes f over X, then x^* minimzes
   $$f_c(x) = f(x) + c \inf_{\overline{x} \in X} ||\overline{x} - x||$$
   over Y.
   (b) Show that if x^* minimizes $f_c(x)$ over Y, then $x^* \in X$, and hence x^* minimizes f over X.