Math4230 Exercise 8

1. Show that x^* is a minimizer of a function f if and only if $0 \in \partial f(x^*)$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a nondecreasing convex function.
   Show that for all $x \in \mathbb{R}$, $g \geq 0$ if $g \in \partial f(x)$.

3. Let $f : \mathbb{R}^n \to (-\infty, \infty]$ be a convex function, let $x \in \text{dom}(f)$. Define
$$f'(x; y) = \inf_{\alpha > 0} \frac{f(x + \alpha y) - f(x)}{\alpha}$$
   Show the following:
   (a) $f'(x; \lambda y) = \lambda f'(x; y)$ for all $\lambda \geq 0$ and $y \in \mathbb{R}^n$.
   (b) $f'(x; .)$ is convex.
   (c) $-f'(x; -y) \leq f'(x; y)$ for all $y \in \mathbb{R}^n$.