Math4230 Exercise 5

1. Let C be a nonempty convex set. Show that $\overline{C} = \overline{\text{ri}(C)}$

2. Let C_1, C_2 be convex sets with $\overline{C_1} = \overline{C_2}$. Show that $\text{ri}(C_1) = \text{ri}(C_2)$.

3. Suppose C_1, C_2 are nonempty convex sets such that $C_1 \subset C_2$.
   (a) Give an example showing that $\text{ri}(C_1)$ may not be a subset of $\text{ri}(C_2)$.
   (b) Suppose $\text{aff}(C_1) = \text{aff}(C_2)$. Show that $\text{ri}(C_1) \subset \text{ri}(C_2)$.

4. Let X be a nonempty convex subset of $\mathbb{R}^n$, let $f : X \to \mathbb{R}$ be a concave function, let
   $$X^* := \{x^* \in X \mid f(x^*) = \inf_{x \in X} f(x)\}$$
   Show that if there exist $x_0 \in X^* \cap \text{ri}(X)$, then f is constant.