THE CHINESE UNIVERSITY OF HONG KONG
MATH 1540 Homework Set 5
Due time 6:30 pm Dec 5, 2016

1. Estimate the value of $e^{0.1}\cos(0.05)$ using the 4-th Taylor polynomial of $h(x,y) = e^x \cos y$ about $(x,y) = (0,0)$.

   Then, show that the error is no more than:

$$\frac{1}{120}\sum_{k=0}^{5} C_k^5 (2)(0.1)^k (0.05)^{5-k}.$$

2. Let $f(x,y) = \dfrac{1}{1+x+y}$.

   (a) Show that the 3-rd Taylor polynomial of f about $(0,0)$ is:

$$T_3(x,y) = \sum_{n=0}^{3}(-1)^n \sum_{k=0}^{n} C_k^n x^k y^{n-k}.$$

   (b) Find a general formula for the n-th Taylor polynomial of f about $(0,0)$, where n is any positive integer.

3. Locate all local maxima, local minima, and saddle points of the following functions (Do not assume all problems must/can be solved using the Second Derivative Test.):

   (a) $f(x,y) = x^3 - y^3 - 2xy - 5$.

   (b) $f(x,y) = \dfrac{1}{1+x^2-y^2}$.

   (c) $f(x,y) = \sqrt[3]{x^2+y^2}$.

4. (a) Show that:

$$\int_{-1}^{2}\int_{3}^{5}\left(x^2y^3 + \frac{x}{y}\right)dy\,dx = 408 + \frac{3}{2}\ln(5/3).$$

   (b) Let:

$$R = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le \pi/2,\ -1 \le y \le 1\}.$$

   Show that:

$$\iint_R xy\cos(2x)\,dA = 0.$$

   (c) Show that:

$$\int_0^2\int_x^2 y^2\cos(xy)\,dy\,dx = \frac{1}{2}(1-\cos 4).$$

(d) Show that:

$$\int_0^3 \int_0^{9-y^2} \frac{ye^x}{9-x}\, dx\, dy = \frac{1}{2}(e^9 - 1).$$

(e) Evaluate:

$$\int_0^1 \int_y^{1-y^2} \int_0^{3-x-y} y\, dz\, dx\, dy.$$