What is a subspace of $\mathbb{R}^n$, in plain words?

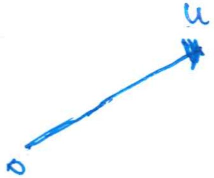
- It is a non-empty collection of vectors in $\mathbb{R}^n$ in which, whenever $u, v, \ldots$ are vectors in this collection, every linear combination of $u, v, \ldots$ will remain in this collection.

  There is 'no chance' of forming a linear combination of vectors in this collection which 'falls outside' this collection.

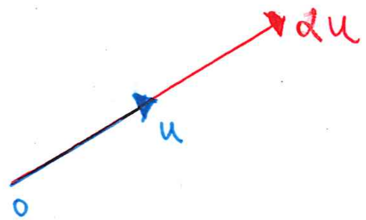
What is a non-subspace of $\mathbb{R}^n$, in plain words?

- It can be the empty set.
- When it is not the empty set, it is a collection of vectors in $\mathbb{R}^n$ in which it 'happens that some linear combination of vectors $u, v, \ldots$ in this collection can be formed in such a way that it 'falls outside' the collection.

One possibility: 'Configuration (M)'.

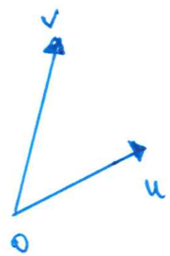


Some u belongs to this collection.

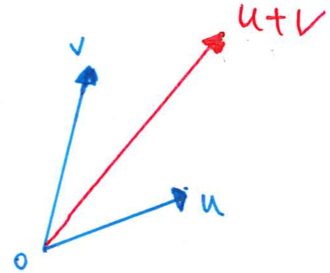


But, for this u, there is some $\alpha \in \mathbb{R}$ for which αu 'falls outside'.

Another possibility: 'Configuration (A)'.



Some u, v belong to this collection.



But, for these u, v, it happens that $u+v$ 'falls outside'.

How to visualize the notion of 'subspace of $\mathbb{R}^n$' in geometric terms?

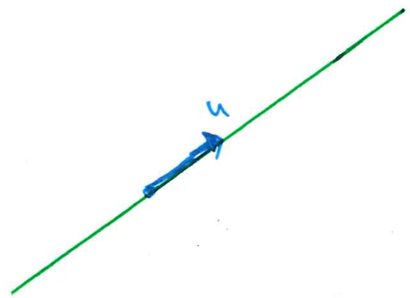
- A subspace of $\mathbb{R}^n$ is a non-empty collection of vectors in $\mathbb{R}^n$ in which there is 'no chance' for 'Configuration (M)' and 'Configuration (A)' to appear.

Subspaces of $\mathbb{R}^2$?

Possibility (1):

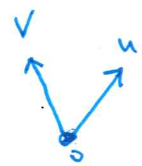
Possibility (2):

Suppose u belongs to this collection.

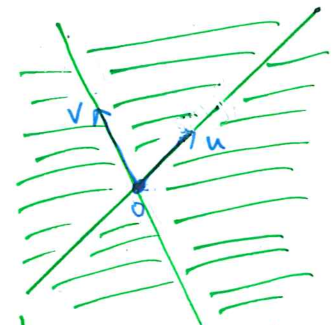


Then every vector whose 'arrow head' lies on the line 'parallel to' u also belongs to this collection.

Possibility (3):



Suppose u, v belong to this collection.

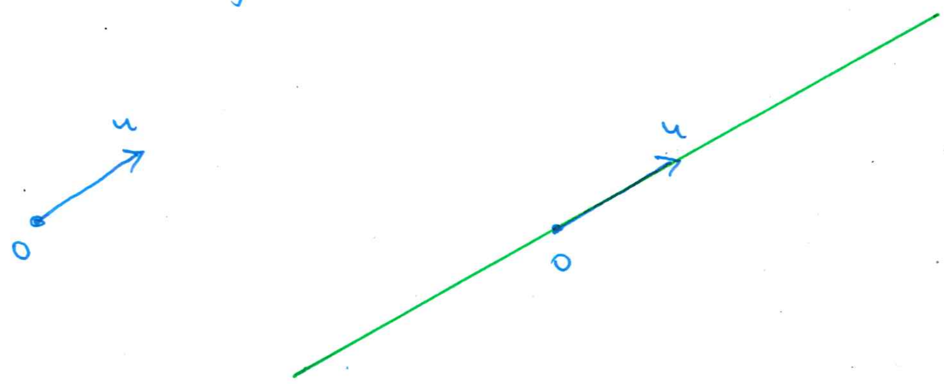


Then every vector in $\mathbb{R}^2$ belongs to this collection.

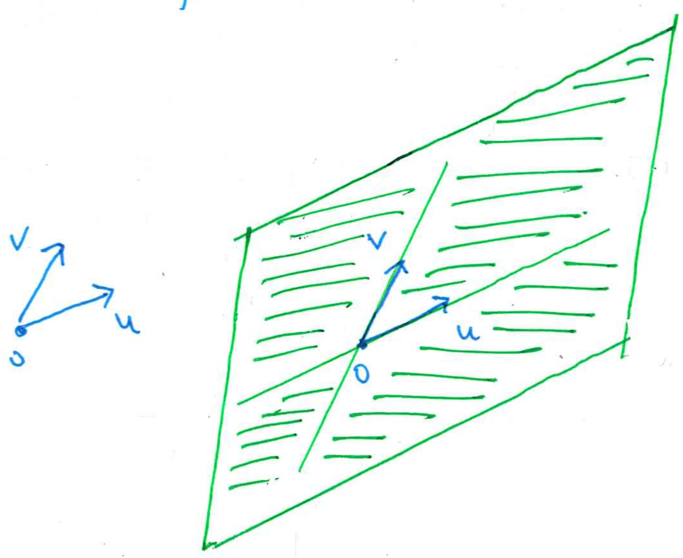
Subspaces of $\mathbb{R}^3$?

Possibility (1):

Possibility (2):



Possibility (3):



Possibility (4):