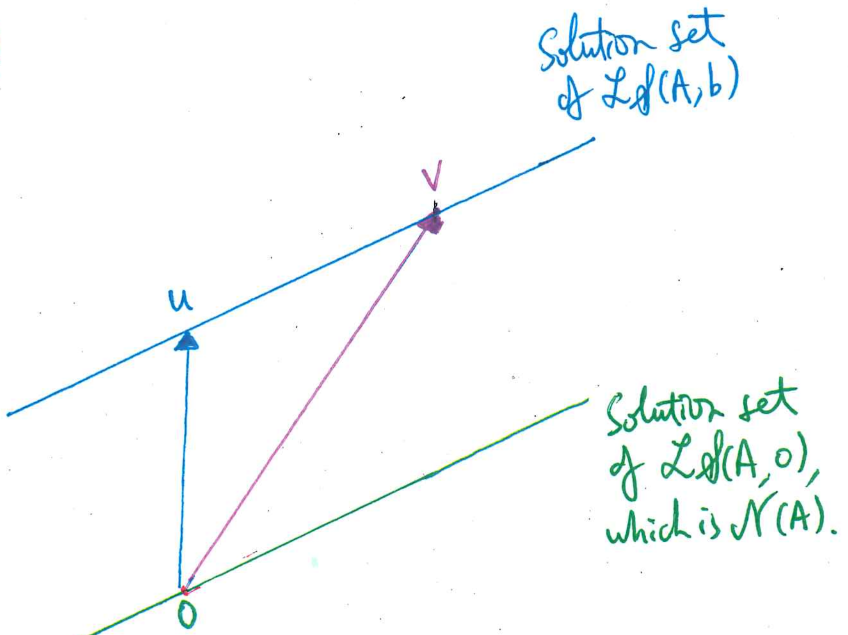
Illustration (1)

Given:

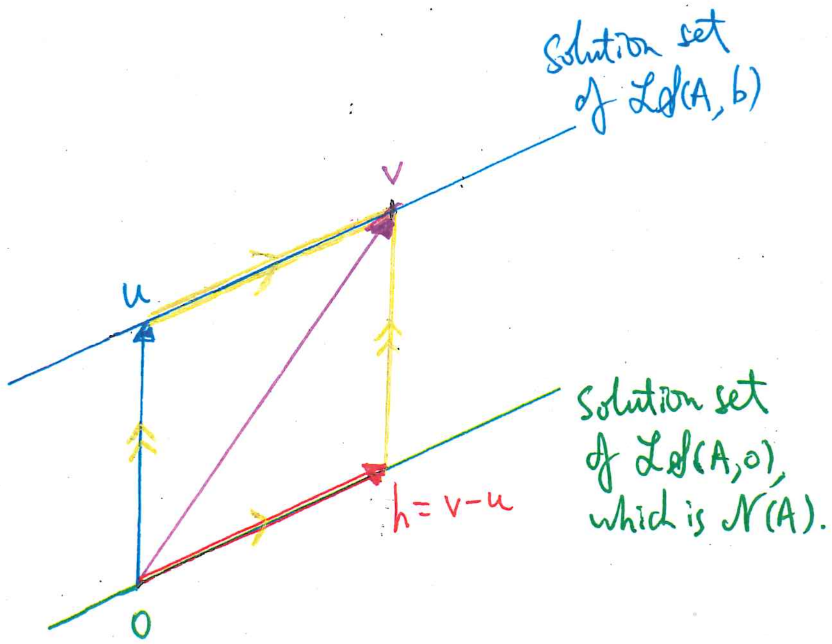
$A = [-1 \quad 2]$, $b \in \mathbb{R}$

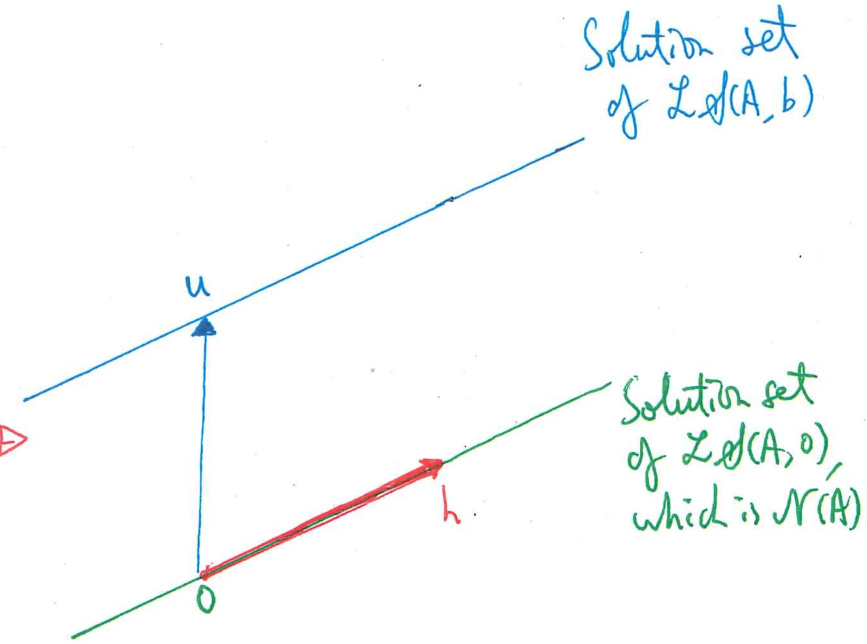
'$x=u$' is any (particular) solution of $\mathcal{LS}(A,b)$,
(for instance, $u = \begin{bmatrix} -b \\ 0 \end{bmatrix}$.)



Suppose '$x=v$' is any one (particular) solution of $\mathcal{LS}(A,b)$.

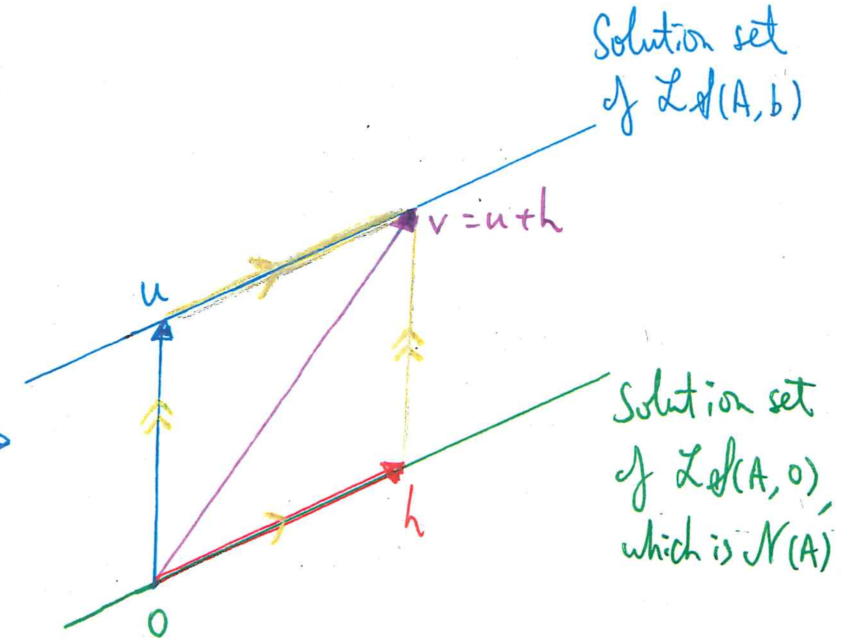
Then, with $h=v-u$, it will happen that '$x=h$' is a solution of $\mathcal{LS}(A,0)$, (or in other words, $h \in \mathcal{N}(A)$).





Suppose '$x=h$' is any one solution of $\mathcal{LS}(A,0)$, (or in other words, $h \in \mathcal{N}(A)$).

Then, with $v=u+h$, it will happen that '$x=v$' is a (particular) solution of $\mathcal{LS}(A,b)$.



Observation:
$\mathcal{N}(A)$ and the solution set of $\mathcal{LS}(A,b)$ are two parallel lines in $\mathbb{R}^2$, the former passing through the point 0, the latter passing through the point u.

Illustration (2).

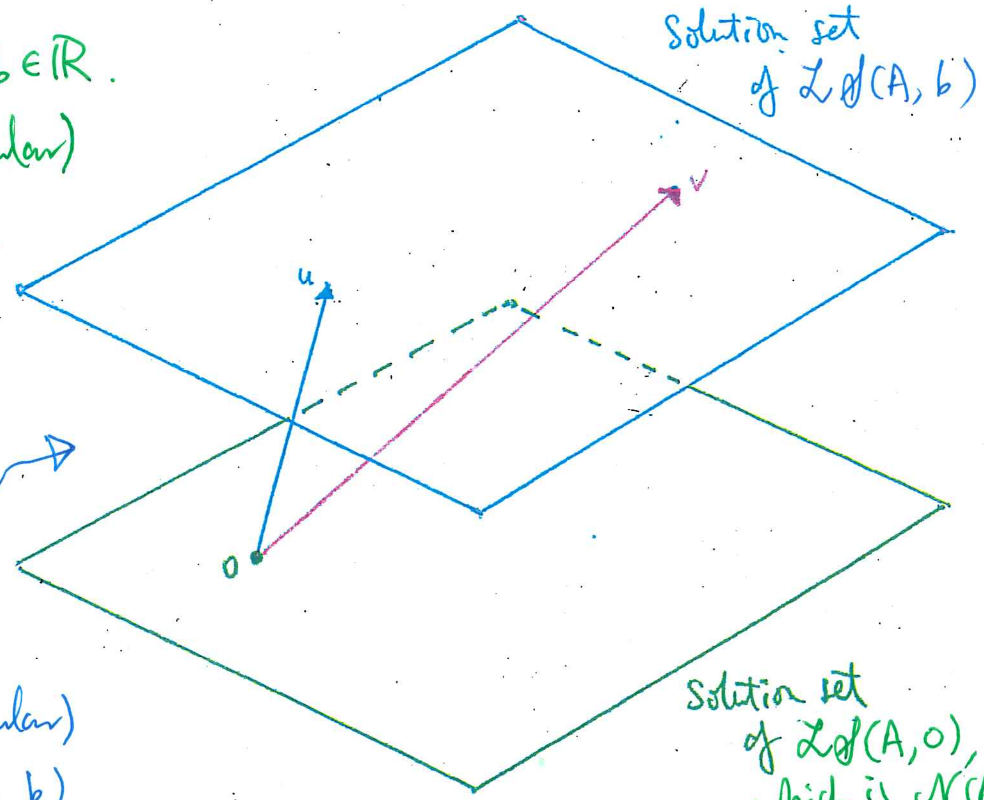
Given:

$A = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & -1 \end{bmatrix}$, $b \in \mathbb{R}$.

'$x = u$' is any (particular) solution of $\mathcal{LS}(A, b)$,

(for instance, $u = \begin{bmatrix} 3b \\ 0 \\ 0 \end{bmatrix}$).

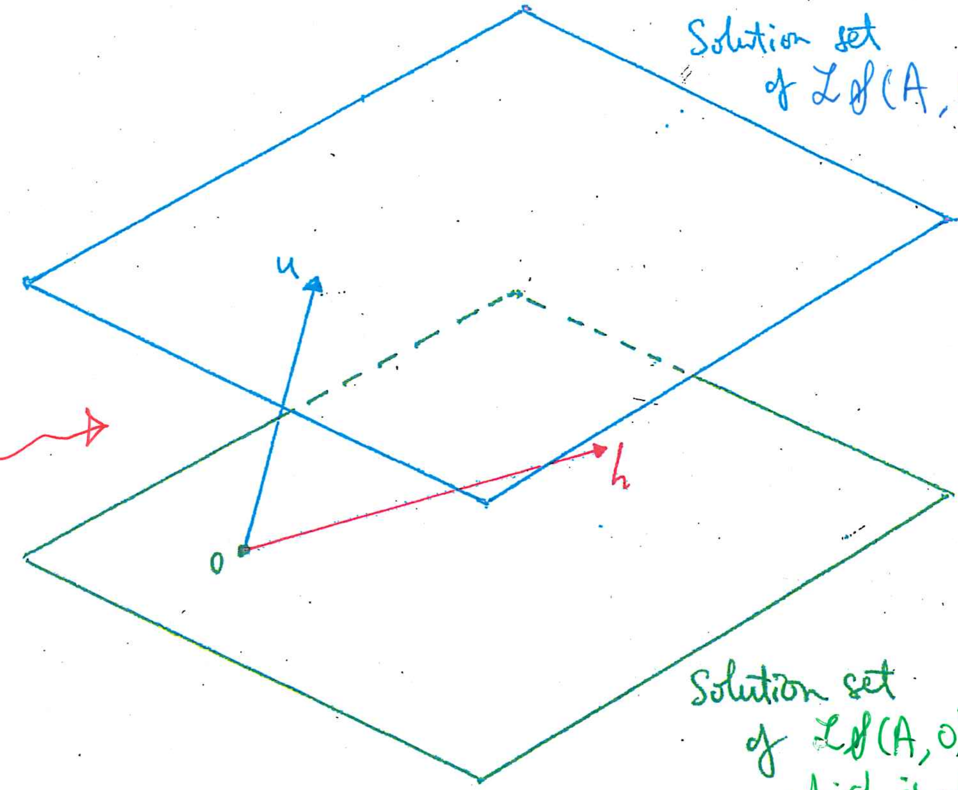
Solution set of $\mathcal{LS}(A, b)$



Suppose '$x = v$' is any one (particular) solution of $\mathcal{LS}(A, b)$.

Solution set of $\mathcal{LS}(A, 0)$, which is $\mathcal{N}(A)$.

Solution set of $\mathcal{LS}(A, b)$

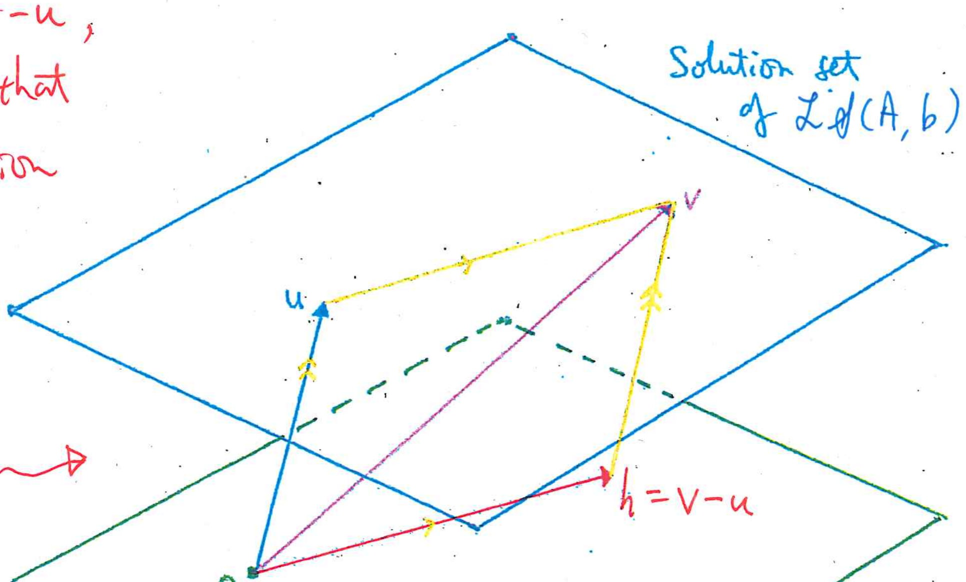


Suppose '$x = h$' is any one solution of $\mathcal{LS}(A, 0)$, (or in other words, $h \in \mathcal{N}(A)$).

Solution set of $\mathcal{LS}(A, 0)$, which is $\mathcal{N}(A)$.

Then, with $h = v - u$, it will happen that '$x = h$' is a solution of $\mathcal{LS}(A, 0)$, (or in other words, $h \in \mathcal{N}(A)$).

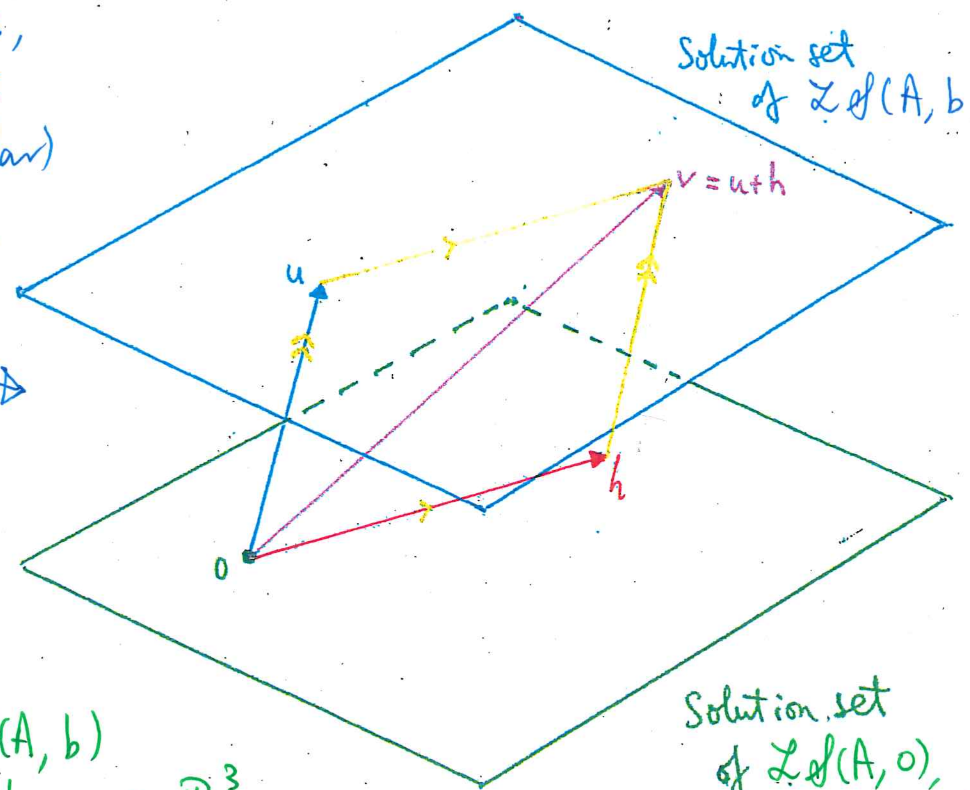
Solution set of $\mathcal{LS}(A, b)$.



Solution set of $\mathcal{LS}(A, 0)$, which is $\mathcal{N}(A)$.

Then, with $v = u + h$, it will happen that '$x = v$' is a (particular) solution of $\mathcal{LS}(A, b)$.

Solution set of $\mathcal{LS}(A, b)$



Solution set of $\mathcal{LS}(A, 0)$, which is $\mathcal{N}(A)$.

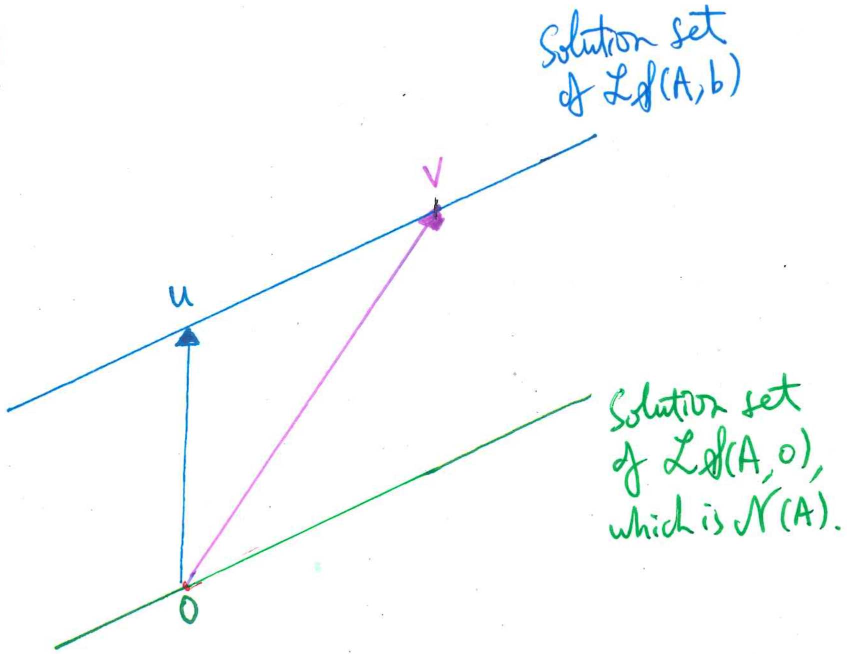
Observation:
$\mathcal{N}(A)$ and the solution set of $\mathcal{LS}(A, b)$ are two parallel planes in $\mathbb{R}^3$, the former passing through the point 0, the latter passing through the point u.

Illustration (3)

Given:

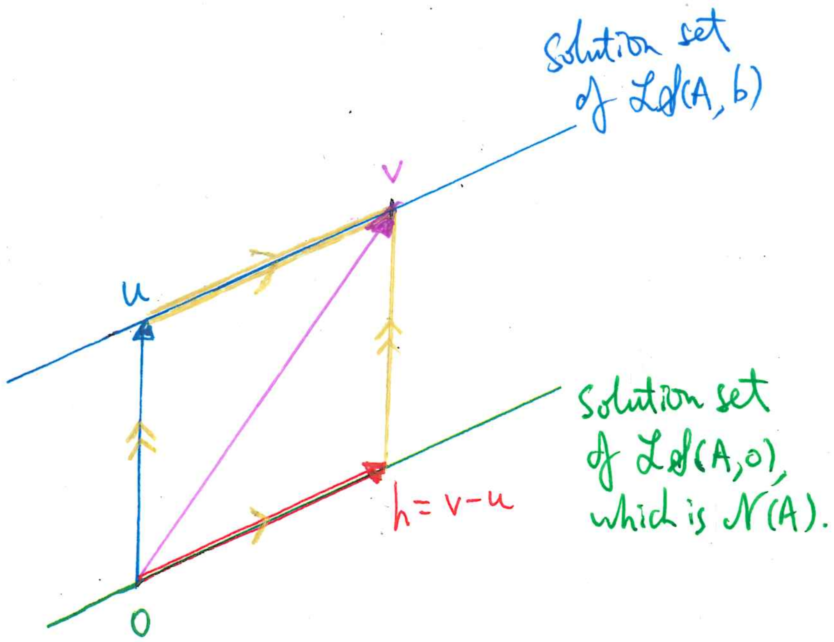
$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbb{R}^2$$

'$x = u$' is any (particular) solution of $\mathcal{LS}(A, b)$, (for instance, $u = \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix}$.)

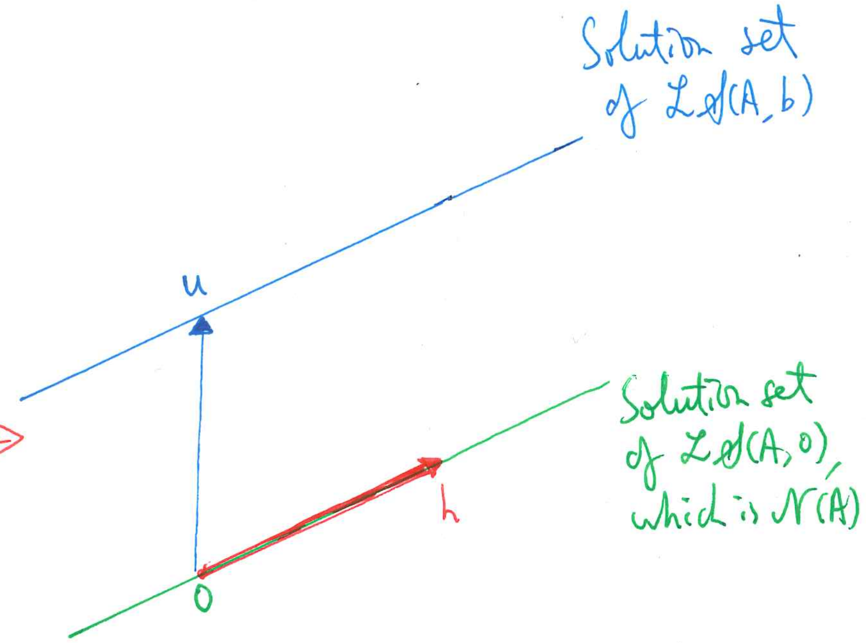


Suppose '$x = v$' is any one (particular) solution of $\mathcal{LS}(A, b)$.

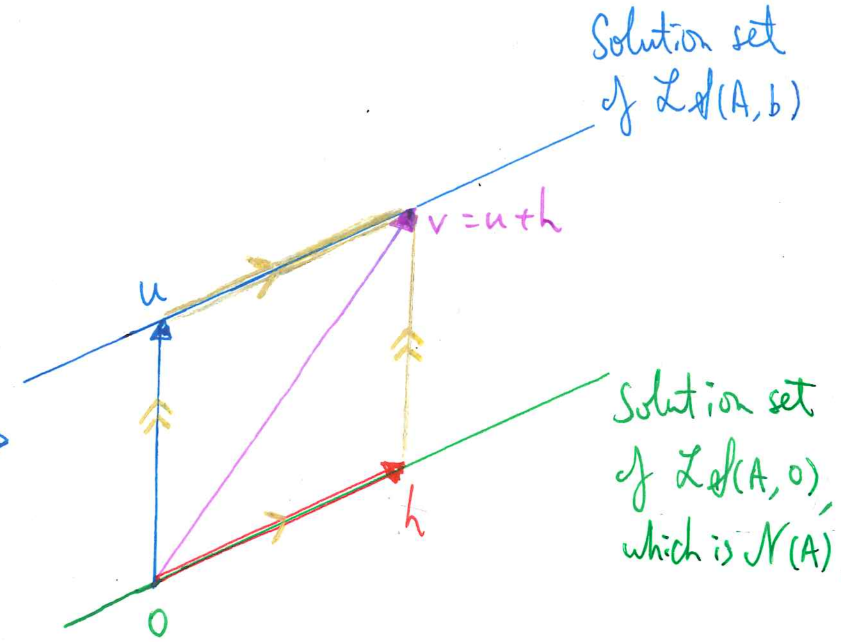
Then, with $h = v - u$, it will happen that '$x = h$' is a solution of $\mathcal{LS}(A, 0)$, (or in other words, $h \in \mathcal{N}(A)$).



Suppose '$x = h$' is any one solution of $\mathcal{LS}(A, 0)$, (or in other words, $h \in \mathcal{N}(A)$).



Then, with $v = u + h$, it will happen that '$x = v$' is a (particular) solution of $\mathcal{LS}(A, b)$.



Observation:

$\mathcal{N}(A)$ and the solution set of $\mathcal{LS}(A, b)$ are two parallel lines in $\mathbb{R}^3$, the former passing through the point 0, the latter passing through the point u.

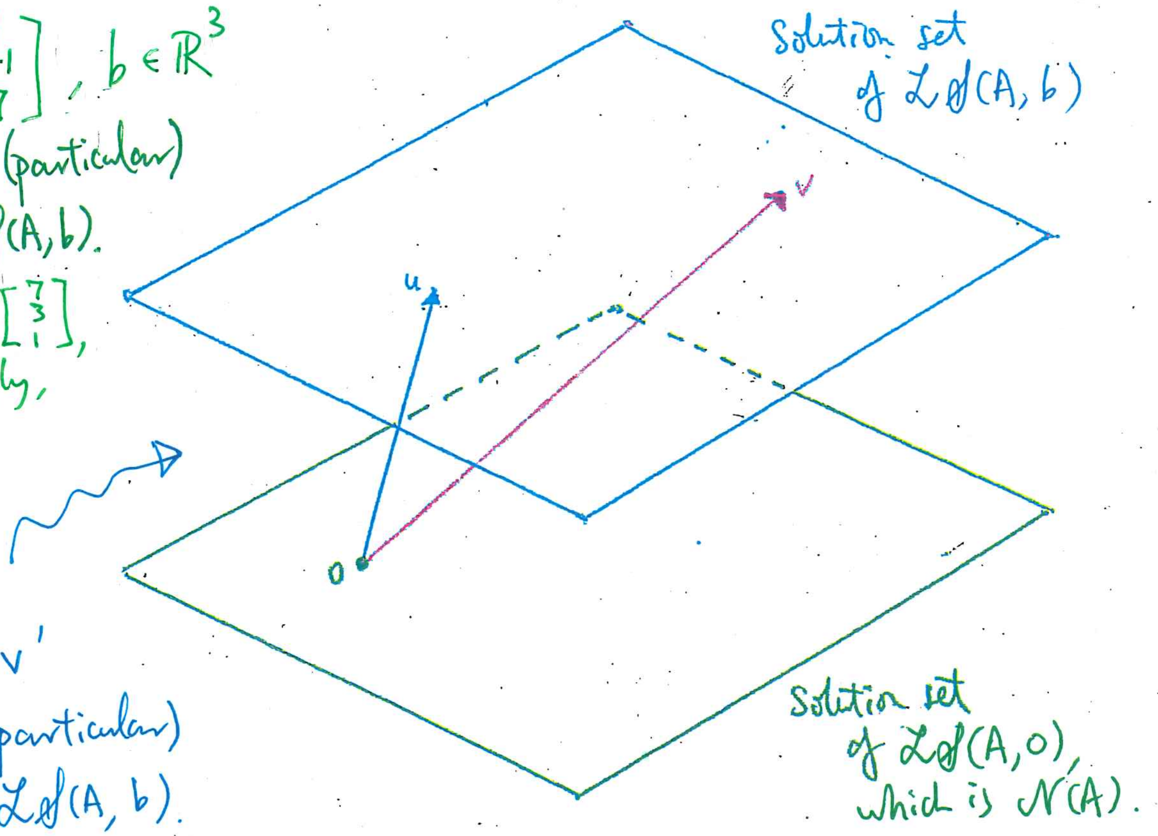
Illustration (4).

Given:

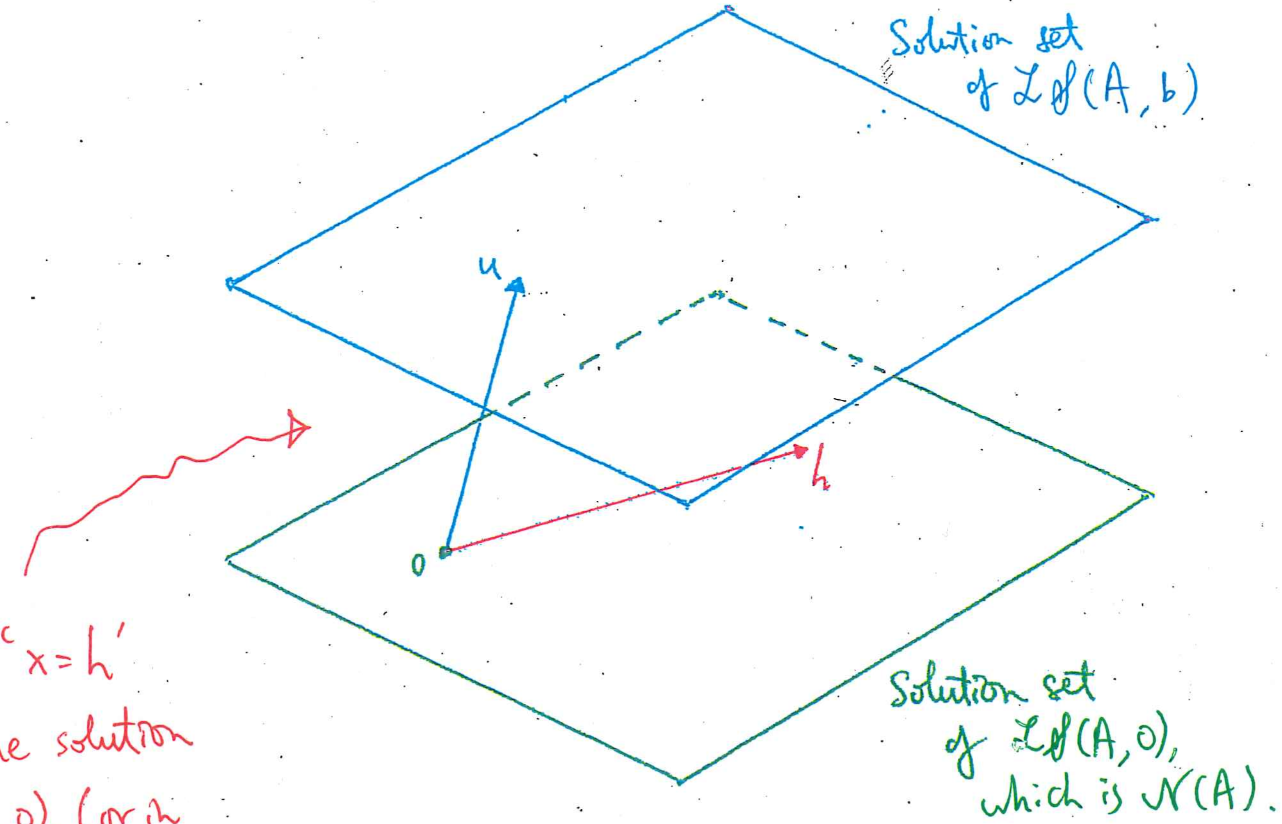
$A=\begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & -1 \\ 3 & 1 & 5 & 7 \end{bmatrix}$, $b \in \mathbb{R}^3$

'$x=u$' is any (particular) solution of $\mathcal{LS}(A,b)$.

(For instance, $b=\begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$, and correspondingly, $u=\begin{bmatrix} -1 \\ 4 \\ 0 \\ 0 \end{bmatrix}$.)

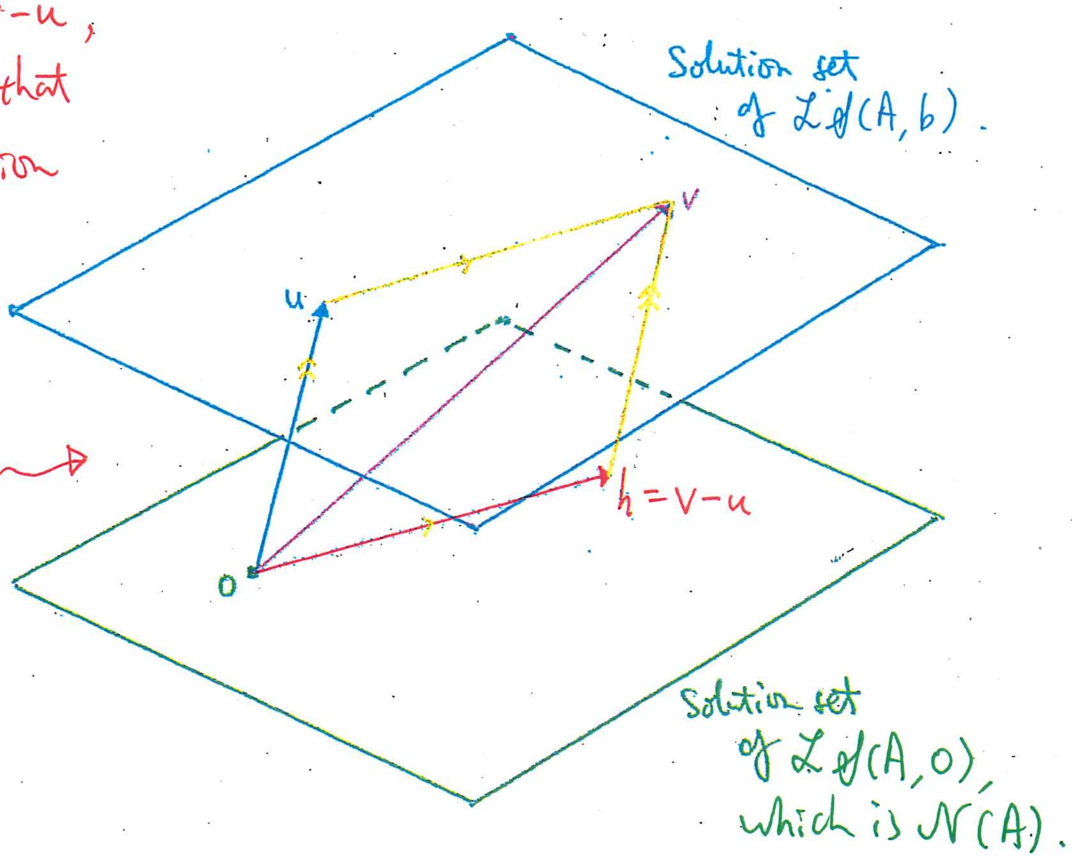


Suppose '$x=v$' is any one (particular) solution of $\mathcal{LS}(A,b)$.

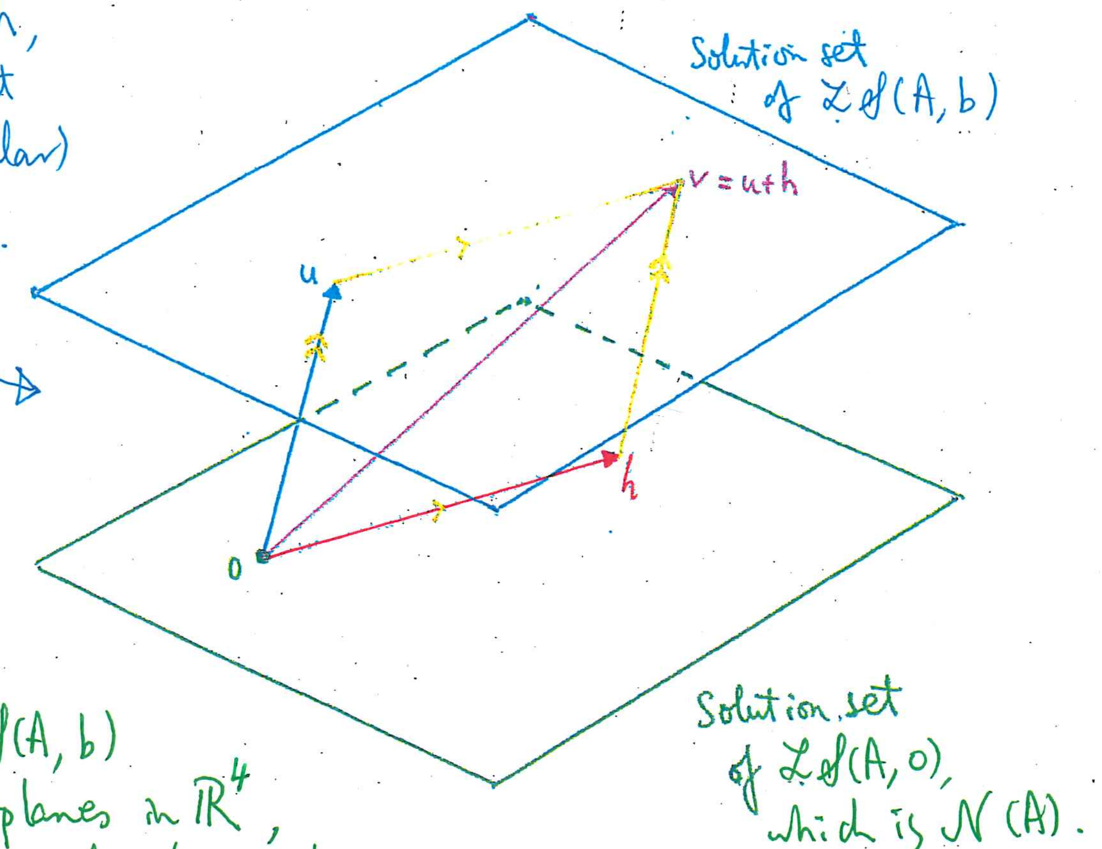


Suppose '$x=h$' is any one solution of $\mathcal{LS}(A,0)$, (or in other words, $h \in \mathcal{N}(A)$).

Then, with $h=v-u$, it will happen that '$x=h$' is a solution of $\mathcal{LS}(A,0)$, (or in other words, $h \in \mathcal{N}(A)$).



Then, with $v=u+h$, it will happen that '$x=v$' is a (particular) solution of $\mathcal{LS}(A,b)$.



Observation:

$\mathcal{N}(A)$ and the solution set of $\mathcal{LS}(A,b)$ are two parallel planes in $\mathbb{R}^4$, the former passing through the point 0, the latter passing through the point u.